Abstract

The equations of motion of a rigid body with an attached spring mass damper are derived. A numerical simulation for the dynamical system is written in MATLAB. The equations of motion are linearized for two different orientations of the damper. The stability conditions are developed from the linearized equations using the Hurwitz-criteria. The conditions for a tuned precession and nutation damper are developed and examined for stability. The conditions for an optimal damper working on earth are found from numerical analysis. A spring-mass damping mechanism for use on the spacecraft simulator is designed and implemented. The functionality of the damper is proven and measurements with different damping fluids are made. The experimental data is compared to the simulation.
Contents

1 Table of symbols .............................................. 6

2 Introduction .................................................... 7

3 Simulation of the Dynamical System ......................... 9
  3.1 System Description ........................................ 9
  3.2 Coordinate Frames and Vector Notation .................. 10
  3.3 Kinematic Equations ...................................... 10
  3.4 Derivation of Dynamical Equations of Motion .......... 11
    3.4.1 Linear Momentum of the Rigid Body ................. 11
    3.4.2 Linear Momentum of the Point Mass ................ 12
    3.4.3 Angular Momentum of the Rigid Body ............... 12
    3.4.4 Angular Momentum of the Point Mass ............... 13
    3.4.5 Derivative of Linear Momentum for the Point Mass . 13
    3.4.6 Derivative of Linear Momentum for the Rigid Body ... 15
    3.4.7 Derivative of Angular Momentum of the Whole System 15
  3.5 Energy Expressions .................................... 16
    3.5.1 Derivation of Kinetic Energy ....................... 16
    3.5.2 Derivation of Potential Energy .................... 17
    3.5.3 Summary of Equations of Motion ................... 18
  3.6 Linearized Equations of Motion ........................ 18
    3.6.1 Horizontal Orientation ............................. 18
    3.6.2 Vertical orientation ............................... 22

4 Setup of Simulation ........................................... 24
  4.1 Description of config.m .................................. 24
  4.2 Description of iohandler.m .............................. 26
  4.3 Description of equofmo.m ............................... 26

5 Stability Analysis of the Linearized Equations of Motion 28
  5.1 Horizontal configuration .................................. 30
  5.2 Vertical Configuration .................................. 32

6 Precession and Nutation of Rigid Bodies ..................... 35
  6.1 Axisymmetric Body ...................................... 37
  6.2 Asymmetric Body ........................................ 39

7 Tuning of the Damper ......................................... 42
  7.1 Vertical Damper .......................................... 42
  7.2 Horizontal Damper ....................................... 43
  7.3 Verification of the Results ............................. 45
  7.4 Selection of Most Effective Design .................... 48
LIST OF FIGURES

List of Figures

3.1 System of rigid body and point mass ........................................... 9
3.2 Free body diagram of the system .................................................. 14
3.3 Horizontal damper orientation ....................................................... 19
3.4 Vertical damper orientation .......................................................... 22
4.5 Flowchart of simulation ............................................................... 24
5.6 Plot of function f ........................................................................... 32
6.7 Plot of angular velocities for an axisymmetric body ......................... 38
6.8 Plot of angular velocities of an asymmetric body ............................ 40
7.9 Plot of angular rates for tuned vertical damper without gravity .......... 46
7.10 Plot of angular rates for detuned vertical damper without gravity .... 46
7.11 Plot of angular rates for tuned horizontal damper without gravity ..... 47
7.12 Plot of angular rates for detuned horizontal damper without gravity ... 47
7.13 Comparison between vertical and horizontal damper, from ref [1] ....... 48
7.14 Plot of spring stiffness (N/m) over damper mass (kg) for horizontal damper, spinrate 75 °/sec ...................................................... 50
7.15 Plot of spring stiffness (N/m) over damper mass (kg) for horizontal damper, spinrate 65 °/sec ...................................................... 50
7.16 Dissipated energy over damping ratio, without gravity .................. 52
7.17 Dissipated energy over Damping ratio, with gravity ....................... 52
7.18 Dissipated energy over damper mass ............................................. 54
7.19 Dissipated energy over distance from center of rotation .................. 54
8.20 Crosssectional view of the damper ............................................... 56
8.21 Linear ball bearing ....................................................................... 57
8.22 Whorl-1 with the damping mechanism .......................................... 58
8.23 Motion of a damped spring mass system, D < 1 ............................... 59
8.24 Setup of damping experiment ......................................................... 61
8.25 Setup of spring test ....................................................................... 64
8.26 Plot of springstiffness over defelction ............................................. 64
9.27 Comparison between the damped and undamped system ................. 65
9.28 Comparison between different oils .................................................. 66
10.29Comparison of the simulation for high viscosity oil with and without friction ........................................................................ 71
10.30Comparison of the simulation for low viscosity oil with and without friction ........................................................................ 72
10.31Comparison of angular rates for different damping fluids with friction .......................................................... 72
A.32 Technical drawings for bearing version, sheet 1 .............................. 87
A.33 Technical drawings for bearing version, sheet 2 .............................. 88
A.34 Technical drawings for bearing version, sheet 3 .............................. 89
A.35 Technical drawings for bearing version, sheet 4 .............................. 90
A.36 Technical drawings for bearing version, sheet 5 .............................. 91
A.37 Technical drawings for bearing version, sheet 6 .............................. 92
A.38 Technical drawings for bearing version, sheet 7 .............................. 93
A.39 Technical drawings for tube version, sheet 1 ................................... 94
A.40 Technical drawings for tube version, sheet 2 ................................... 95
A.41 Technical drawings for tube version, sheet 3 ................................. 96
A.42 Technical drawings for tube version, sheet 4 ................................. 97
A.43 Technical drawings for tube version, sheet 5 ................................. 98
A.44 Technical drawings for tube version, sheet 6 ................................. 99
A.45 Technical drawings for tube version, sheet 7 ................................. 100
A.46 Measurement for spinrate -60°/sec ............................................. 102
A.47 Measurement for spinrate -70°/sec ............................................. 102
A.48 Measurement for spinrate -75°/sec ............................................. 103
A.49 Measurement for spinrate 60°/sec .............................................. 103
A.50 Measurement for spinrate 70°/sec .............................................. 104
A.51 Measurement for spinrate 75°/sec .............................................. 104
A.52 Comparison of damping fluids, spinrate -60°/sec ......................... 106
A.53 Comparison of damping fluids, spinrate -70°/sec ......................... 106
A.54 Comparison of damping fluids, spinrate -75°/sec ......................... 107
A.55 Comparison of damping fluids, spinrate 60°/sec ......................... 107
A.56 Comparison of damping fluids, spinrate 70°/sec ......................... 108
A.57 Comparison of damping fluids, spinrate 75°/sec ......................... 108
List of Tables

4.1 Overview over simulation parameters ........................................ 25
7.2 Spring stiffness for tuned dampers .......................................... 44
7.3 Approximate design parameter ................................................ 49
8.4 Overview of used oils ............................................................ 58
8.5 Needed values for damping experiments ..................................... 61
8.6 Summary of experimental results, low viscosity ............................ 62
8.7 Summary of experimental results, high viscosity ............................ 62
A.8 Testresults for spring 1 .......................................................... 78
A.9 Testresults for spring 2 .......................................................... 79
A.10 Testresults for oil with low viscosity ......................................... 79
A.11 Testresults for oil with high viscosity ........................................ 80
1 Table of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>denotes the rigid body</td>
<td>-</td>
</tr>
<tr>
<td>( B )</td>
<td>reaction force in the pedestal</td>
<td>N</td>
</tr>
<tr>
<td>( b )</td>
<td>distance of dampermass in nominal position from CR</td>
<td>m</td>
</tr>
<tr>
<td>( c )</td>
<td>spring stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>( d )</td>
<td>damping coefficient</td>
<td>N sec/m</td>
</tr>
<tr>
<td>( E )</td>
<td>unity matrix</td>
<td>1</td>
</tr>
<tr>
<td>( F_{\text{con}} )</td>
<td>constraint forces acting on the dampermass</td>
<td>N</td>
</tr>
<tr>
<td>( F_d^* )</td>
<td>forces acting on the dampermass (spring, damping, constraint)</td>
<td>N</td>
</tr>
<tr>
<td>( F_{g,B} )</td>
<td>gravity force acting on the CM of the rigid body</td>
<td>N</td>
</tr>
<tr>
<td>( F_{g,P} )</td>
<td>gravity force acting on the dampermass</td>
<td>N</td>
</tr>
<tr>
<td>( F_{Th} )</td>
<td>Thruster force</td>
<td>N</td>
</tr>
<tr>
<td>( g )</td>
<td>gravitational acceleration</td>
<td>m/sec(^2)</td>
</tr>
<tr>
<td>( H )</td>
<td>angular momentum vector of the total system</td>
<td>N m sec</td>
</tr>
<tr>
<td>( H_B )</td>
<td>angular momentum vector of the rigid body</td>
<td>N m sec</td>
</tr>
<tr>
<td>( H_P )</td>
<td>angular momentum vector of the dampermass</td>
<td>N m sec</td>
</tr>
<tr>
<td>( I )</td>
<td>inertia matrix of the whole system</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>( I_s )</td>
<td>inertia matrix of the simulator</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>( I_d )</td>
<td>inertia matrix of the dampermass</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>( I_{dn} )</td>
<td>inertia matrix of the dampermass</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>( M_{\text{ext}} )</td>
<td>external torques</td>
<td>N m</td>
</tr>
<tr>
<td>( M )</td>
<td>mass of the rigid body</td>
<td>kg</td>
</tr>
<tr>
<td>( m )</td>
<td>dampermass</td>
<td>kg</td>
</tr>
<tr>
<td>( \hat{n} )</td>
<td>unit vector of damper alignment</td>
<td>1</td>
</tr>
<tr>
<td>( P )</td>
<td>denotes the point mass</td>
<td>-</td>
</tr>
<tr>
<td>( p )</td>
<td>linear momentum vector of the whole system</td>
<td>N sec</td>
</tr>
<tr>
<td>( p_B )</td>
<td>linear momentum vector of the rigid body</td>
<td>N sec</td>
</tr>
<tr>
<td>( p_d )</td>
<td>linear momentum vector of the dampermass</td>
<td>N sec</td>
</tr>
<tr>
<td>( p_{dn} )</td>
<td>linear momentum of the dampermass in ( \hat{n} )</td>
<td>N sec</td>
</tr>
<tr>
<td>( q )</td>
<td>quaternion vector with first three components</td>
<td>1</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>quaternion vector with all four components</td>
<td>1</td>
</tr>
<tr>
<td>( q_4 )</td>
<td>scalar component of quaternions</td>
<td>1</td>
</tr>
<tr>
<td>( R )</td>
<td>inertial position vector of point 0</td>
<td>m</td>
</tr>
<tr>
<td>( R_{bn} )</td>
<td>direction cosine matrix from {n}- to {b}- frame</td>
<td>m</td>
</tr>
<tr>
<td>( r_{0} )</td>
<td>nominal position vector of the mass from point O</td>
<td>m</td>
</tr>
<tr>
<td>( r_{om,B} )</td>
<td>position vector of CM of rigid body from point O</td>
<td>m</td>
</tr>
<tr>
<td>( r_{th} )</td>
<td>moment arm of thruster from point O</td>
<td>m</td>
</tr>
<tr>
<td>( T )</td>
<td>kinetic energy</td>
<td>J</td>
</tr>
<tr>
<td>( u )</td>
<td>deviation of dampermass along ( \hat{n} )</td>
<td>m</td>
</tr>
<tr>
<td>( v )</td>
<td>velocity of dampermass along ( \hat{n} )</td>
<td>m/sec</td>
</tr>
<tr>
<td>( \omega )</td>
<td>angular velocity of the {b}- to {n}- frame</td>
<td>rad/sec</td>
</tr>
</tbody>
</table>
Spin stabilization was one of the first attitude control concepts because it is a relatively easy and inexpensive way to keep a spacecraft oriented in a certain direction relative to an inertial frame. Pertubations however can force the spacecraft into precession where the spin axis of the spacecraft performs a coning motion. This motion causes a misalignment of the spacecraft’s spin axis with respect to its desired orientation and must be reduced. Instead of using the spacecraft’s actuators to control and minimize the coning motion, a precession or nutation damper can be used to let the spacecraft return to a one axis spin almost autonomously. One of the most famous cases in which the attitude of a spacecraft was influenced by damping is Explorer 1, the first satellite of the United States. Explorer 1 was equipped with several radial antennas, which were flexible rods. The deflection of the antennas caused energy dissipation and since the satellite was spinning around its minor axis, it lost its original attitude after a few orbits and ended up in a major axis spin. Motivated by this experience, more research was done in this area to find different mechanisms which use energy dissipation for attitude stabilization. Since the early days of space travel different kinds of damping mechanisms were developed and made as effective as possible. In the following, a short overview over the most important concepts is given.

As already mentioned, the simplest form of a nutation damper is a rod, which is rigidly attached to a spacecraft (ref [2]). Energy is dissipated through deflection of the rod or through magnetic hysteresis losses in earth’s magnetic field. The energy dissipation rate is relatively low, which results in a low pointing accuracy. A more sophisticated type of damper using viscous damping is the ring nutation damper. This damper consists of a hollow ring totally or partially filled with a fluid. The viscous forces in the fluid and between the fluid and the inner area of the ring dissipate energy (ref [3]). These dampers were flown on the spacecraft IMAGE (ref [4]) and the spaceprobe HELIOS (ref [5]). Other types of damper dissipate energy due to the damped motion of a mass relative to the damper cage. Two different main categories which a great variety of different realizations can be found: spring-mass dampers and pendulum dampers. The latter type can for example consist of a masspendulum which is coupled through torsion springs to its cage. If the damper undergoes a periodic excitation, the mass starts to oscillate. Energy can be damped out by filling the cage with a fluid (ref [5]). The concept of the spring mass damper also shows a variety of different realizations. The configuration often found in literature consists of a mass which is attached with a spring to its cage and moves linearly through a fluid (ref [5]). The damper is placed with an offset to the satellite’s spin axis and is oriented either parallel or normal to it. The first case is commonly referred to as a precession damper while the latter case is called nutation damper. Reference [1] and reference [6] examine and compare the effectivity of precession and nutation dampers. In P. C. Hughes’ book (ref [5]) a rigid body in space with an attached precession damper is investigated with respect to stability and tuning. In reference [7] the motion of a rigid body with an attached precession damper is analyzed in cases where the stability criteria are not satisfied. The equilibria of a rigid body with a point-mass damper and
a reaction wheel are examined in reference [8]. Spring mass nutation dampers were used on the satellites SYNCOM and Explorer 18 (ref [5]).

In this research project a damping mechanism is developed for the use on the Distributed Spacecraft Attitude Control System Simulator (DSACSS) which is located at the campus of the Virginia Tech under the direction of Prof. Hall. The DSACSS consists of two independent satellite simulators, Whorl-I and Whorl-II. Whorl-I is a tabletop platform, mounted on an air bearing, which allows a rotation of ±5° around the pitch and roll axes and a full rotation around the yaw-axis. The simulator is controlled by a PC/104+ form-factor computer, whose operational software is written in C++. A wireless LAN connection is used for communication between both simulators and the external computers in the lab. The simulator has two different types of attitude control devices for three-axis control: a compressed air thruster system and three momentum wheels. Attitude determination is provided by three-axis accelerometers and rate gyros. The system’s center of gravity can be moved in three dimensions by linear actuators. The second simulator, Whorl-II, is mounted on an air bearing in a dumbbell configuration, which allows a full rotation around the yaw and roll axes and ±30° of freedom in pitch. It has equivalent equipment to Whorl-I.

The dynamics of the simulator and a spacecraft are not the same due to their different gravitational environment. A satellite in an earth orbit always rotates around its center of mass. The simulator however always rotates around a fixed point since it is mounted on an air bearing. In order to make both systems comparable, the center of mass of the simulator is adjusted to be close to the center of rotation so that gravitational torques are minimized. If a spring mass damper is implemented, the collective center of gravity of the simulator and the damper changes due to the movement of the damper mass and gravitational torques occur. Furthermore the mass on the simulator is also affected by gravity. Therefore different stability conditions for the damper are expected.

In this research project it will be shown, how a spring mass damping system for the use on a simulator can be realized. Therefore the equations of motion for the system are developed and a MATLAB-simulation is written. The stability conditions for certain orientations of the damper are developed from the linearized equations of motion. It is examined how the performance of the damper can be maximized. The most promising design for the damping mechanism is realized and implemented on Whorl-1. Finally, the functionality of the damper is proven and the measurements are compared to the simulation.
3 Simulation of the Dynamical System

In the following chapters the equations of motion are derived for the system consisting of the simulator with the attached spring mass damper. The equations of motion are derived in the most general form to keep the simulation as flexible as possible. Later for the stability analysis the equation are simplified with loss of generality for special cases.

3.1 System Description

The dynamical system consists of the simulator and the attached damper as depicted in figure (3.1). The simulator is assumed to be a rigid body $B$. The rigid body has a body-fixed coordinate frame $\{b\}$ attached to it. The damper mass is treated as a point mass $P$, which is connected with a spring to the rigid body and it is allowed to move along $\hat{n}$. Note that the bracket to mount the damper on the simulator and all other parts of the structure of the damper excluding the mass contribute to mass properties of the rigid body. The damping is assumed to be linear. The base point is chosen to be point $O$, which is the center of rotation CR (i.e. the hemispherical air bearing) of the simulator. Since the simulator is stationary, $O$ is assumed to be inertially fixed. Note that the center of mass CM of the total system changes due to the movement of the mass $P$.

Figure 3.1: System of rigid body and point mass
3.2 Coordinate Frames and Vector Notation

Two different coordinate frames are used for the derivation of the equations of motion. The first coordinate frame is an inertial coordinate frame (or Newtonian coordinate frame) \( \{n\} \). A vector expressed in that coordinate frame consists of the components \( \mathbf{x} = x\hat{n}_1 + y\hat{n}_2 + z\hat{n}_3 \). The second coordinate frame is a body fixed coordinate frame \( \{b\} \). This coordinate frame is attached to the body and follows its translational as well as rotational movement. A vector in the \( \{b\} \)-frame will be expressed in the following components: \( \mathbf{x} = u\hat{b}_1 + v\hat{b}_2 + w\hat{b}_3 \).

3.3 Kinematic Equations

The description of the attitude of the rigid-body/damper system is necessary to include the effect of environmental torques on the system, which is in our case the gravity force. Furthermore the attitude is needed to find expressions for the potential energy of the system.

The attitude description of one coordinate frame towards another can be expressed using an EULER-angle sequence. The disadvantage of these parameters is a singularity which occurs in the kinematic equations for certain angles. Since the equations of motion need to be developed for arbitrary rotation, the use of EULER angles is not recommended.

Fortunately other ways to describe the orientation of a body can be found. All of them are based on Euler’s theorem, which states that every rotation of a rigid body around a fixed point can be completely described by only one rotation around a fixed axis by a certain angle. This axis is called 'Euler axis' or 'Eigenaxis' and is further denoted as \( \mathbf{a} \). The angle is known as 'Eulerangle' and referred to as \( \Phi \). The rotation cosine matrix can also be expressed in terms of Euler axis and Eulerangle (ref [9], chapter 3, page 11).

With \( \mathbf{a} \) and \( \Phi \), four new variables, the so called quaternions, can be defined:

\[
\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \sin\left(\frac{\Phi}{2}\right),
\]

\[
q_4 = \cos\left(\frac{\Phi}{2}\right). \tag{3.1}
\]

It is also possible to find an expression for the direction cosine matrix in terms of quaternions (ref [9], chapter 3, page 12):

\[
\mathbf{R}_{BN} = (q_1^2 - \mathbf{q}^T \mathbf{q}) \cdot \mathbf{E} + 2(\mathbf{qq}^T - q_4 \mathbf{q}^\times). \tag{3.2}
\]

Note that in equation (3.2) \( \mathbf{E} \) stands for the unity matrix. The operation \( \mathbf{q}^\times \) is herein
3 SIMULATION OF THE DYNAMICAL SYSTEM

defined as

\[ q^* = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}. \]

With the definition \( \bar{q} = \begin{bmatrix} q^T \\ q_4 \end{bmatrix} \), we can also express the kinematic equation for \( \omega \) in terms of quaternions (Ref. [10], chapter 3, pages 95-100):

\[ \dot{\bar{q}} = \frac{1}{2} \begin{bmatrix} q^* + q_4E \\ -q^T \end{bmatrix} \cdot \omega. \] (3.3)

This equation has the advantage of being free of singularities and is therefore very suitable for arbitrary rotations.

3.4 Derivation of Dynamical Equations of Motion

Similar equations of motion for a rigid body with an attached spring mass damper are derived as a textbook example in Hughes book (Ref. [5]). Hughes studies an inertially moving body and uses a base point which is both center of mass and center of rotation. Therefore the resulting equations show some differences. To simplify a comparison a very similar notation as used in Hughes was also chosen for this derivation.

3.4.1 Linear Momentum of the Rigid Body

The linear momentum of a rigid body is the integral of the inertial velocity of all infinitesimal mass pieces of the body

\[ p_B = \int_B \dot{r}dm = \int_B (\dot{\bar{R}} + \dot{r})dm \] (3.4)

where \( \dot{r} \) is the inertial postion vector of a masspiece \( dm \), which can be split up into the inertial position vector \( \bar{R} \) of the base point \( O \) and the vector \( r \) from the base point \( O \) to the infinitesimal mass pieces. Since the \( O \) is a stationary point in this case, \( \dot{\bar{R}} \) is zero. With the transport theorem we can find the inertial derivative of \( \dot{r} \):

\[ \dot{r} = \dot{B} \frac{d}{dt} r + \omega \times r = \omega \times r. \] (3.5)

The derivative of \( r \) with respect to \( \{ b \} \) is zero, since the body is rigid. Substituting the equation above into (3.4) yields

\[ p_B = M(\omega \times r_{cm,B}) \] (3.6)

with the mass of the rigid body \( M \) and the angular velocity \( \omega \) of the rigid body towards the inertial frame.
3.4.2 Linear Momentum of the Point Mass

Analog to the derivation above the linear momentum of the point mass can be found. We start with the definition of the position vector of the point mass:

\[ \mathbf{r}_d = \mathbf{r}_{d0} + u \mathbf{n}, \quad (3.7) \]

where \( u \) is a scalar coordinate to measure the deviation of \( m \) and \( \mathbf{r}_{d0} \) the initial position of the mass. Taking the derivative of \( \mathbf{r}_d \) with the transport theorem yields:

\[ \dot{\mathbf{r}}_d = N \frac{d}{dt} \mathbf{r}_d = B \frac{d}{dt} \mathbf{r}_d + \omega \times \mathbf{r}_d. \quad (3.8) \]

In this case the body frame derivative of the position vector \( \mathbf{r}_d \) of the mass is not zero, since \( \mathbf{P} \) is allowed to move along \( \mathbf{n} \). We find:

\[ N \frac{d}{dt} \mathbf{r}_d = \dot{u} \mathbf{n} + \omega \times \mathbf{r}_d. \quad (3.9) \]

The linear momentum equation can be written as mass times inertial velocity:

\[ \mathbf{p}_d = m(\dot{\mathbf{R}} + \dot{\mathbf{r}}_d). \quad (3.10) \]

The derivative of \( \mathbf{R} \) is zero again and \( \dot{\mathbf{r}}_d \) was already found above. We find the linear momentum:

\[ \mathbf{p}_d = m(\dot{u} \mathbf{n} + \omega \times \mathbf{r}_d). \quad (3.11) \]

Of special interest is furthermore the linear momentum of the mass in the direction of \( \mathbf{n} \). Therefore equation (3.11) is multiplied with \( \mathbf{n} \):

\[ \mathbf{p}_{d,n} = m \dot{u} \mathbf{n} + \omega \times (\mathbf{M} \mathbf{r}_{cm,B} + \mathbf{m} \mathbf{r}_d) \quad (3.12) \]

The skewmatrix notation \( \mathbf{r}^\times \) is herein defined as

\[ \mathbf{r}^\times = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}. \]

The linear momentum of the rigid body and the point mass can be found as the sum of the individual linear momenta:

\[ \mathbf{p} = \mathbf{p}_B + \mathbf{p}_d = M(\omega \times \mathbf{r}_{cm,B}) + m(\dot{u} \mathbf{n} + \omega \times \mathbf{r}_d) \]
\[ = m \dot{u} \mathbf{n} + \omega \times (\mathbf{M} \mathbf{r}_{cm,B} + m \mathbf{r}_d). \quad (3.13) \]

3.4.3 Angular Momentum of the Rigid Body

The angular momentum over the base point \( \mathbf{O} \) of the rigid body is the body integral of the cross product of the position vector with respect to \( \mathbf{O} \) and the inertial velocity vector over all infinitesimal mass pieces.

\[ \mathbf{H}_B = \int_B (\mathbf{r} \times \dot{\mathbf{r}}) dm. \quad (3.14) \]
Again with the identity $\dot{\mathbf{r}} = \mathbf{\omega} \times \mathbf{r}$ we find:

$$H_B = \int_B (\mathbf{r} \times (\mathbf{\omega} \times \mathbf{r}))d\mathbf{m} = -\int_B \mathbf{r} \times \mathbf{r} \times \mathbf{\omega} d\mathbf{m}. \quad (3.15)$$

$$H_B = \int_B \left[ \begin{array}{ccc}
r_x^2 & r_x r_y & r_x r_z \\
r_y r_x & r_y^2 & r_y r_z \\
r_x r_z & r_y r_z & r_z^2 \end{array} \right] \mathbf{\omega} d\mathbf{m} = I_B \mathbf{\omega} \quad (3.16)$$

In the equation above $I_B$ is the inertia matrix of the rigid body evaluated over the base point $O$.

### 3.4.4 Angular Momentum of the Point Mass

The angular momentum of the point mass can be found from:

$$H_B = (\mathbf{r}_d \times m\dot{\mathbf{r}}_d) \quad (3.17)$$

The inertial derivative of $\mathbf{r}_d$ was already found in equation (3.8):

$$H_P = m(\mathbf{r}_d \times (\dot{\mathbf{u}} \hat{n} + \mathbf{\omega} \times \mathbf{r}_d)) = m(\mathbf{r}_d \times \dot{\mathbf{u}} \hat{n} - \mathbf{r}_d \times (\mathbf{r}_d \times \mathbf{\omega})), \quad (3.18)$$

$$H_P = m\mathbf{r}_d \times \dot{\mathbf{u}} \hat{n} + I_d \mathbf{\omega}. \quad (3.19)$$

In this equation $I_d = -m\mathbf{r}_d^2 \dot{\mathbf{r}}_d^2$ is the inertia matrix of the point mass evaluated over the base point $O$.

The angular momentum vector of the total system can be found by adding equations (3.16) and (3.20)

$$H = H_B + H_P = (I_B + I_d)\mathbf{\omega} + m\dot{\mathbf{u}}(\mathbf{r}_d \times \hat{n}) = I\mathbf{\omega} + m\dot{\mathbf{u}}(\mathbf{r}_d \times \hat{n}). \quad (3.21)$$

The inertia matrix $I$ consists of the contributions of the rigid body and the damper mass. For applications with rigid bodies only it is convenient to express all vectors and the inertia matrices with respect to the body-fixed frame, since in this case the inertia matrices are constant. Though we did not yet pick a coordinate frame, it is noteworthy that $I$ still depends on time if evaluated in the $\{b\}$-frame, since $m$ moves with respect to this frame and changes the mass properties of the whole system.

### 3.4.5 Derivative of Linear Momentum for the Point Mass

According to Newton’s second law, the derivative of the linear momentum is equal to the sum of the applied external forces. The forces acting on the point mass, after it was cut free from the rigid body as shown in figure (3.2), are the gravity force $\mathbf{F}_{g,P}$ and the forces caused by the damping mechanism $\mathbf{F}_{d}^*$:

$$\frac{d}{dt} \mathbf{p}_d = \sum \mathbf{F}_{ext} = \mathbf{F}_{g,P} + \mathbf{F}_{d}^*. \quad (3.22)$$
\( \mathbf{F}_d^* \) contains the spring force \( F_s = cu \), the damping force \( F_{damp} = d\dot{u} \), both of which act parallel to \( \hat{n} \), as well as the constraint forces \( \mathbf{F}_{\text{con}} \), which force the mass to move only along \( \hat{n} \):

\[
\mathbf{F}_d^* = (-cu - d\dot{u})\hat{n} + \mathbf{F}_{\text{con}} \tag{3.23}
\]

Figure 3.2: Free body diagram of the system

Together with equation (3.22) we can find

\[
\frac{\mathcal{N}}{d} \frac{d}{dt} \mathbf{p}_d = \mathbf{F}_{g,P} - cu\hat{n} - d\dot{u}\hat{n} + \mathbf{F}_{\text{con}} \tag{3.24}
\]

Since the linear momentum is a vector, it can be derived according to the transport theorem:

\[
\frac{\mathcal{N}}{d} \frac{d}{dt} \mathbf{p} = B \frac{d}{dt} \mathbf{p} + \omega \times \mathbf{p}, \tag{3.25}
\]

\[
B \frac{d}{dt} \mathbf{p} = \frac{\mathcal{N}}{d} \frac{d}{dt} \mathbf{p} - \omega \times \mathbf{p} \tag{3.26}
\]

With equation (3.26) the inertial based derivative of the linear momentum can now be expressed as derivates with respect to the body frame. To avoid confusion, the \( \{b\} \)-frame derivative is denoted \( B \frac{d}{dt} \mathbf{p} = \dot{\mathbf{p}} \).

The derivative of \( \mathbf{p}_d \) for the point mass is according to equation (3.24):

\[
B \dot{\mathbf{p}}_d = -\omega \times \mathbf{p}_d + \mathbf{F}_{g,P} - cu\hat{n} - d\dot{u}\hat{n} + \mathbf{F}_{\text{con}} \tag{3.27}
\]
Again we are especially interested in the derivative of the linear momentum in \( \hat{n} \)-direction. Equation (3.27) multiplied with \( \hat{n} \) yields:

\[
\begin{align*}
B \dot{p}_{d,n} &= \hat{n} \cdot B \dot{p}_d = \hat{n} \cdot ( -\omega \times p_B + F_{g,P} - cu\hat{n} - d\dot{u}\hat{n} + F_{\text{con}} ) \\
&= \hat{n} \cdot B p_d - \hat{n} \cdot ( -\omega \times p_d ) + \hat{n} \cdot F_{g,P} - cu - d\dot{u}.
\end{align*}
\] (3.28)

Since \( F_{\text{con}} \) is the constraint force, which avoids a movement of \( m \) in another direction except for \( \hat{n} \), it does not act in the \( \hat{n} \)-direction and \( \hat{n} \cdot F_{\text{con}} = 0 \). Therefore we find for \( B p_{d,n} \)

\[
B p_{d,n} = \hat{n} \cdot B p_d = \hat{n} \cdot ( -\omega \times p_d ) + \hat{n} \cdot F_{g,P} - cu - d\dot{u}.
\] (3.29)

### 3.4.6 Derivative of Linear Momentum for the Rigid Body

The derivative of the linear momentum of the rigid body alone is

\[
N \frac{d}{dt} p_B = B + F_{g,B} + F_{Th} - F_d^* \] (3.30)

with the reaction in the pedestal \( B \), the gravity force on the rigid body \( F_{g,B} \), and the thrust force \( F_{Th} \).

This inertial derivative can also be transformed using equation (3.26):

\[
B \dot{p}_B = -\omega \times p_B + B + F_{g,B} + F_{Th} \] (3.31)

Now we are able to express the derivative of the total linear momentum of the system by adding equation (3.31) and (3.27):

\[
B \dot{p} = -\omega \times p + B + F_{g,B} + F_{g,P} + F_{Th} \] (3.32)

At this point the use of equation (3.32) needs to be discussed. Since the system is stationary it has only four degrees of freedom. Three for the rotation of the rigid body and one for the translational movement of the mass. We will later see that the motion of the system can be fully described with the three rotational and the one translational equations of motion which we are about to develop. One would expect the momentum equation (3.13) for the whole system to be equal to zero, since it is not undergoing a translational movement in the inertial frame. The reason why it is not zero, is that the common center of mass actually moves inertially caused by the displacement of the mass and the rotation of the body. The momentum equation for the whole system can therefore be used to calculate the inertial velocity of the common center of mass which is not of interest in this case. Also equation (3.32) can be omitted since it only provides information about the force \( B \) acting on the pedestal. Therefore equations (3.32) and (3.31) are not longer used.

### 3.4.7 Derivative of Angular Momentum of the Whole System

The inertial derivative of angular momentum is equal to the applied external torques:

\[
\dot{H} = \sum M_{\text{ext}} = r_{cm} \times F_{g,B} + r_d \times F_{g,P} + r_{th} \times F_{Th}.
\] (3.33)

The torques acting on the total system are the torques causes by the force of gravity and a troque causes by the thrusters. \( r_{th} \) is in this case the momentum arm of the thruster
force $F_{th}$. Similar to the linear momentum, the angular momentum is expressed in a body frame derivative as well.

$$B\dot{H} = -\omega \times H + r_{cm} \times F_{g,B} + r_d \times F_{g,P} + r_{th} \times F_{th}.$$  \hfill (3.34)

### 3.5 Energy Expressions

In order to provide information about the effectiveness of the damper, the development of the system’s energy must be studied. The total energy of the system consists of the kinetic energy of the rigid body and the point mass as well as the potential energy which changes if the center of mass of the simulator and the point mass move through the local gravity field. Additionally potential energy is stored in the spring if the mass is deviated from its nominal position.

#### 3.5.1 Derivation of Kinetic Energy

The most general expression for kinetic energy is

$$T = \frac{1}{2} \int_B \dot{\rho} \cdot \dot{\rho} dm$$  \hfill (3.35)

with the inertial position vector $\rho = R + r$. For the rigid body we already used equation (3.5) to find that $\dot{\rho} = \omega \times r$:

$$T_B = \frac{1}{2} \int_B (\omega \times r) \cdot (\omega \times r) dm$$  \hfill (3.36)

With the identity $(a \times b) \cdot c = (a \cdot (b \times c))$ this equation can be rewritten as

$$T_B = \frac{1}{2} \int_B (\omega \cdot (r \times (\omega \times r))) dm$$  \hfill (3.37)

We have already seen in equation (3.15) that the term $(r \times (\omega \times r)) = I_s \omega$. This leads to:

$$T_B = \frac{1}{2} \omega \cdot I_s \cdot \omega.$$  \hfill (3.38)

The kinetic energy for the point mass can be calculated using similar identities as above:

$$T_d = \frac{1}{2} m \dot{r}_d \cdot \dot{r}_d = \frac{1}{2} m (\dot{u} \mathbf{n} + \omega \times r_d) \cdot (\dot{u} \mathbf{n} + \omega \times r_d),$$  \hfill (3.39)

$$T_d = \frac{1}{2} m (\dot{u}^2 + 2 \dot{u} \mathbf{n} \cdot (\omega \times r_d)) + \frac{1}{2} m (\omega \times r_d) \cdot (\omega \times r_d),$$  \hfill (3.40)

$$T_d = \frac{1}{2} m \dot{u}^2 + \frac{1}{2} \omega \times I_d \cdot \omega + m \dot{u} (r_d \times \mathbf{n}).$$  \hfill (3.41)

The total kinetic energy can be easily found by adding equation (3.38) and equation (3.41):

$$T = \frac{1}{2} \omega I_s \cdot \omega + \frac{1}{2} m \dot{u}^2 + \frac{1}{2} \omega \cdot I_d \cdot \omega + m \dot{u} (r_d \times \mathbf{n}),$$

$$T = \frac{1}{2} \omega \cdot I \cdot \omega + \frac{1}{2} m \dot{u}^2 + m \dot{u} (r_d \times \mathbf{n}).$$  \hfill (3.42)
Another important value is the rate of the potential energy $\dot{T}$ of the total system. The following expression for the kinetic energy of the rigid body can be found from the derivative of equation (3.38):

$$\dot{T}_B = \frac{1}{2} \dot{\omega} \cdot I_s + \frac{1}{2} \omega \cdot I_s \cdot \dot{\omega} = \omega \cdot I_s \cdot \dot{\omega}. \quad (3.43)$$

With the identity $\dot{\mathbf{H}} = \mathbf{I} \cdot \dot{\omega}$ this expression is reduced to

$$\dot{T}_B = \dot{\mathbf{H}} \cdot \omega = \mathbf{M}_{ext} \cdot \omega, \quad (3.44)$$

with $\mathbf{M}_{ext}$ as the sum of all external torques.

$$\dot{T}_B = (\mathbf{r}_{cm,B} \times \mathbf{F}_{g,B} - \mathbf{r}_d \times \mathbf{F}_d + \mathbf{r}_{th} \times \mathbf{F}_Th) \cdot \omega, \quad (3.45)$$

A similar derivation is made for the rate of the kinetic energy of the point mass damper:

$$\dot{T}_d = N \frac{d}{dt} \left( \frac{1}{2} m \mathbf{r}_d \cdot \dot{\mathbf{r}}_d \right) = \dot{\mathbf{r}}_d m \left( \frac{d}{dt} \mathbf{r}_d \right) = \dot{\mathbf{r}}_d \sum \mathbf{F}_{ext}. \quad (3.46)$$

The external forces acting on the point mass are the damper forces $\mathbf{F}_d$ and the gravity force $\mathbf{F}_{g,P}$.

Using equation (3.23) we can find

$$\dot{T}_d = (\dot{\mathbf{u}} \hat{\mathbf{n}} + \omega \times \mathbf{r}_d) \cdot (\mathbf{F}_{g,P} + \mathbf{F}_d^s), \quad (3.47)$$

$$\dot{T}_d = \dot{\mathbf{u}} \hat{\mathbf{n}} \cdot \mathbf{F}_{g,P} + \mathbf{F}_{g,P} \cdot (\omega \times \mathbf{r}_d) + \dot{\mathbf{u}} \hat{\mathbf{n}} \cdot \mathbf{F}_d^s + \mathbf{F}_d^s \cdot (\omega \times \mathbf{r}_d), \quad (3.48)$$

$$\dot{T}_d = \omega \cdot (\mathbf{r}_d \times \mathbf{F}_d^s) + \omega \cdot (\mathbf{r}_d \times \mathbf{F}_{g,P}) + \dot{\mathbf{u}} (-\mathbf{c}_u - \mathbf{d} \dot{\mathbf{u}}) + \dot{\mathbf{u}} \mathbf{F}_{g,P} \cdot \hat{\mathbf{n}}. \quad (3.49)$$

The total kinetic energy rate is finally found from the summation of equation (3.49) and equation (3.45):

$$\dot{T} = \omega \cdot (\mathbf{r}_{cm,B} \times \mathbf{F}_{g,B} + \mathbf{r}_d \times \mathbf{F}_{g,P} + \mathbf{r}_{th} \times \mathbf{F}_Th) + \dot{\mathbf{u}} (-\mathbf{c}_u - \mathbf{d} \dot{\mathbf{u}}) + \dot{\mathbf{u}} \mathbf{F}_{g,P} \cdot \hat{\mathbf{n}}. \quad (3.50)$$

### 3.5.2 Derivation of Potential Energy

The term for the kinetic energy is more straightforward to find. According to $V = mgh$ for the potential energy of the center of mass and $V = \frac{1}{2} cu^2$ for the energy stored in the spring we find:

$$V = M g \mathbf{i}_3 \cdot (\mathbf{R} + \mathbf{r}_{cm,B}) + m g \mathbf{i}_3 \cdot (\mathbf{R} + \mathbf{r}_d) + \frac{1}{2} cu^2, \quad (3.51)$$

where $\mathbf{R}$ is the position vector of the center of rotation $O$ and $\mathbf{r}_{cm,B}$ and $\mathbf{r}_d$ are the vectors from $O$ to the center of mass of $B$ and the point mass, respectively.

Taking the time derivative of the previous equation yields the potential energy rate. Note that $M g \mathbf{i}_3 = -\mathbf{F}_{g,B}$ and $m g \mathbf{i}_3 = -\mathbf{F}_{g,P}$. We find with $\dot{\mathbf{R}} = 0$, $\dot{\mathbf{r}}_{cm,B} = \omega \times \mathbf{r}_{cm,B}$ and $\dot{\mathbf{r}}_d = \dot{\mathbf{u}} \hat{\mathbf{n}} + \omega \times \mathbf{r}_{cm,B}$:

$$\dot{V} = -\omega \cdot (\mathbf{r}_{cm,B} \times \mathbf{F}_{g,B}) - \dot{\mathbf{u}} \mathbf{F}_{g,P} \cdot \hat{\mathbf{n}} - \omega (\mathbf{r}_{cm,B} \times \mathbf{F}_{g,P}) + cu \dot{u}, \quad (3.52)$$

Finally the total energy rate can be found from $\dot{E} = \dot{T} + \dot{V}$:

$$\dot{E} = \omega (\mathbf{r}_{th} \times \mathbf{F}_{Th}) - d \dot{u}^2. \quad (3.53)$$

It becomes obvious that thrusting and all other external torques change the total energy of the system. The effect of the damping is obvious in the term $d \dot{u}^2$. 
3.5.3 Summary of Equations of Motion

For a better overview, the derived equations of motion for the system are summarized. We end up with two equations describing the translational movement of the damper-mass, the linear momentum equations for \( p_{d,n} \) and \( \dot{p}_{d,n} \) and two equations describing the rotational movement of the total system, which are the angular momentum equations \( H \) and \( \dot{H} \). All equations are cross-coupled since the movement of the pointmass affects the rigid body and vice versa.

\[
p_{d,n} = (m \dot{u} - m \hat{n}^T r_d \times \omega), \tag{3.54}
\]

\[
\dot{p}_{d,n} = \hat{n} \cdot (-\omega \times p_d) + \hat{n} \cdot F_{g,P} - cu - d \dot{u}, \tag{3.55}
\]

\[
H = I \omega + m \dot{u} (r_d \times \hat{n}), \tag{3.56}
\]

\[
\dot{H} = -\omega \times H + r_{cm} \times F_{g,B} + r_d \times F_{g,P} + r_{th} \times F_{Th}. \tag{3.57}
\]

3.6 Linearized Equations of Motion

For a later stability analysis the equations of motion need to be linearized. To simplify the calculations, the position and orientation of the damper with respect to the rigid body are limited to certain orientations now, with loss of generality. Furthermore we assume that no thrust is applied. Note that all vectors are expressed in the body-fixed-coordinate frame, therefore the index is omitted. Vectors expressed in a different coordinate frame will carry an index.

3.6.1 Horizontal Orientation

We first consider a vertical orientation of the damper. That means the damper lies in the \( \hat{b}_1 - \hat{b}_2 \)-plane as depicted in figure (3.3). The nominal position of the mass is directly on the negative \( \hat{b}_1 \)-axis with a certain distance \( b \) from the center of rotation. Therefore we set

\[
r_{d0} = (-b, 0, 0)^T \text{ and } r_{CM,B} = \left( \frac{m}{M} b, 0, 0 \right)^T
\]

The normal vector \( \hat{n} \) which describes the orientation of the damper points in the \( \hat{b}_2 \)-direction:

\[
\hat{n} = (0, 1, 0)^T.
\]

The position of the damper mass with respect to the body frame can therefore be found as

\[
r_d = (-b, u, 0)^T. \tag{3.58}
\]

The attitude description of the rigid body is done using a \((1,2,3)\)-EULER angle sequence with the three angles \( \alpha = (\alpha_1, \alpha_2, \alpha_3) \). The rotation matrix from the inertial-frame to the body-frame is according to this choice:

\[
R_{bn} = \begin{bmatrix}
  c\alpha_2 c\alpha_3 & c\alpha_2 s\alpha_3 & -s\alpha_2 \\
  -c\alpha_1 s\alpha_3 + s\alpha_1 c\alpha_2 c\alpha_3 & c\alpha_1 c\alpha_3 + s\alpha_1 s\alpha_2 s\alpha_3 & s\alpha_1 c\alpha_2 \\
  s\alpha_1 s\alpha_3 + c\alpha_1 s\alpha_2 c\alpha_3 & -s\alpha_1 c\alpha_3 + c\alpha_1 s\alpha_2 s\alpha_3 & c\alpha_1 c\alpha_2
\end{bmatrix} \tag{3.59}
\]
Figure 3.3: Horizontal damper orientation

Linearizing this equation for small angles yields

\[
\tilde{R}_{bn} = \begin{bmatrix} 1 & \alpha_3 & -\alpha_2 \\ -\alpha_3 & 1 & \alpha_1 \\ \alpha_2 & -\alpha_1 & 1 \end{bmatrix} = E - \alpha^\times. \tag{3.60}
\]

The simulator is furthermore assumed to spin around its \(\hat{b}_3\)-axis with a certain angular velocity \(\nu\) and the \(\hat{b}_3\)-axis is assumed to be closely aligned to the inertial \(\hat{b}_3\)-axis. Therefore a linearized expression for the angular velocity in terms of \(\nu\) and \(\alpha\) can be found.

\[
\tilde{\omega} = (\dot{\alpha}_1 - \alpha_2 \nu, \dot{\alpha}_2 + \alpha_1 \nu, \dot{\alpha}_3 + \nu) \tag{3.61}
\]

At last a linearized expression for the gravity force in the \(\{b\}\)-frame needs to be found. The gravity force vector is always pointing in the negative \(\hat{n}_3\)-direction. This vector can be rotated with the linearized rotation matrix into the body frame.

\[
\tilde{F}_G = m_i g \tilde{R}_{bn} \cdot (0, 0, -1)^T = m_i g (\alpha_2, -\alpha_1, -1)^T \tag{3.62}
\]

where \(m_i\) is the appropriate mass, in this case the damper mass or the simulator mass. Having all these expressions together we can find a linearized equation for the linear momentum of the damper mass:

\[
\tilde{p}_d = m(\dot{u}\hat{n} + \tilde{\omega} \times r_d) \tag{3.63}
\]

Putting the definitions above into equation (3.63) yields:

\[
\tilde{p}_d = m \begin{pmatrix} -uv \\ -b\dot{\alpha}_3 - \nu b + \dot{u} \\ b\dot{\alpha}_2 - b\nu \alpha_1 \end{pmatrix}. \tag{3.64}
\]
For later use the time derivative of this equation with respect to the body frame is taken:

\[ \dot{\tilde{p}}_d = m \begin{pmatrix} -\dot{\omega} v \\ -b\dot{\alpha}_3 + \ddot{u} \\ b\dot{\alpha}_2 - b\dot{\alpha}_1 \end{pmatrix}. \] (3.65)

Now, equation (3.28) for the derivative of the linear momentum in the \( \hat{n} \)-direction needs to be linearized:

\[ \dot{\tilde{p}}_{d,n} = \hat{n} \cdot ( -\tilde{\omega} \times \tilde{p}_d ) + \hat{n} \cdot \tilde{F}_{g,P} - cu - d\dot{u} \] (3.66)

Making use of the definitions for the gravity force and equation (3.64) yields the following result:

\[ \dot{\tilde{p}}_{d,n} = mu^2 - cu - d\dot{u} - m\dot{\alpha}_1. \] (3.67)

If we now combine equation (3.67) and the second component of equation (3.65) we find the linearized equation of motion for the damper mass:

\[ m\ddot{u} - mb\ddot{\alpha}_3 - mu^2 + cu + d\dot{u} + m\dot{\alpha}_1 = 0. \] (3.68)

The next step is to linearize the two equations for the angular momentum. Therefore a linearized version of the timevariant inertia matrix must be found. We can split the inertia matrix up into the constant part for the rigid body and a varying part for the point mass. We already found an expression for the inertia matrix of the damper for the derivation of equation (3.15):

\[ I_d = -mr_d^x r_d^x \]

Therefore we can find

\[ I = I_{sim} - mr_d^x r_d^x = I_{sim} + m \begin{pmatrix} u^2 & bu & 0 \\ bu & b^2 & 0 \\ 0 & 0 & u^2 + b^2 \end{pmatrix}. \] (3.69)

The constant terms are then grouped together with \( I_1 = I_{sim,1}, I_2 = I_{sim,2} + mb^2, I_3 = I_{sim,3} + mb^2 \) and non-linear terms are omitted:

\[ \tilde{I} = \begin{pmatrix} I_1 & mbu & 0 \\ mbu & I_2 & 0 \\ 0 & 0 & I_2 \end{pmatrix}. \] (3.70)

Now, the linearized version of the angular momentum vector \( \tilde{H} \) can be found from equation (3.21):

\[ \tilde{H} = \tilde{I} \cdot \tilde{\omega} + m\dot{u}(r_d \times \hat{n}) = \begin{pmatrix} I_1(\dot{\alpha}_1 - \alpha_2\nu) \\ I_2(\dot{\alpha}_2 + \alpha_2\nu) \\ I_3(\dot{\alpha}_3 + \nu) - mb\dot{u} \end{pmatrix}. \] (3.71)

Again for later use we take the time derivative of the angular momentum vector in the body-fixed frame:

\[ \dot{\tilde{H}} = \begin{pmatrix} I_1(\ddot{\alpha}_1 - \dot{\alpha}_2\nu) \\ I_2(\ddot{\alpha}_2 + \dot{\alpha}_2\nu) \\ I_3\ddot{\alpha}_3 - mb\ddot{u} \end{pmatrix}. \] (3.72)
At last a linearized version of the derivative of the total angular momentum given in equation (3.21) needs to be found. It is helpful to develop the expression for the torques caused by gravity force first. With the use of equation (3.62) we can find:

\[ r_{cm} \times F_{g,B} = M g \left( \frac{m_b}{M}, 0, 0 \right)^T \times (\alpha_2, -\alpha_1, -1)^T = (0, -m_b g, m_b \nu_1)^T \] (3.73)

\[ r_d \times F_{g,P} = m g \left( -b, u, 0 \right)^T \times (\alpha_2, -\alpha_1, -1)^T = (-u m g, -m_b g, m_b \nu_1)^T \] (3.74)

Finally the derivative of the angular momentum vector can be found using the previous results:

\[ B \dot{\tilde{H}} = -\tilde{\omega} \times \tilde{H} + r_{cm} \times F_{g,B} + r_d \times F_{g,P} \] (3.75)

\[ B \dot{\tilde{H}} = \begin{pmatrix} (I_3 - I_2)(\dot{\alpha}_2 \nu - \alpha_1 \nu^2) - m g u \\ (I_1 - I_3)(\dot{\alpha}_1 - \alpha_2 \nu^2) \\ 0 \end{pmatrix} \] (3.76)

It is interesting to note that the gravitational torques remaining in the equation only act around the \( b_1 \)-axis. This is due to the fact that the center of mass of the rigid body is located on the opposite side of the center of rotation as the nominal position of the damper mass. As a consequence all other torques cancel out. With the help of equation (3.72) the following three linearized equations can be found:

\[ I_1(\dot{\alpha}_1 - \dot{\alpha}_2 \nu) + (I_3 - I_2)(\dot{\alpha}_2 \nu - \alpha_1 \nu) + m g u = 0 \]
\[ I_2(\dot{\alpha}_2 + \dot{\alpha}_2 \nu) + (I_1 - I_3)(\dot{\alpha}_1 - \alpha_2 \nu) = 0 \]
\[ I_3 \ddot{\alpha}_3 - m_b u = 0 \] (3.77)

For a better overview and understanding equations (3.77) and (3.68) can be grouped together in matrix form. We define the vector \( q = (\alpha_1, \alpha_2, \alpha_3, u) \) and find the following equation:

\[ M \cdot \ddot{q} + (G + D) \dot{q} + K \cdot q = 0 \] (3.78)

with the following definitions for the matrices:

\[ M = \begin{bmatrix} I_1 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & I_3 & -m_b \\ 0 & 0 & -m_b & m \end{bmatrix}, \] (3.79)

\[ G = (I_3 - I_2 - I_1) \nu \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \] (3.80)

\[ D = \text{diag}\{0,0,0,d\}, \] (3.81)

\[ K = \begin{bmatrix} (I_3 - I_2) \nu^2 & 0 & 0 & m g \\ 0 & (I_3 - I_1) \nu^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m g & 0 & 0 & c - m \nu^2 \end{bmatrix}. \] (3.82)
3.6.2 Vertical orientation

The second case of interest is the vertical installation of the damper. That means the vector \( \hat{n} \) points into the \( \hat{b}_3 \)-direction and the damper mass is located on the negative \( \hat{b}_1 \)-axis again as depicted in figure (3.4).

![Figure 3.4: Vertical damper orientation](image)

The position vectors for the center of mass of the rigid body and the damper mass are:

\[
\mathbf{r}_{d0} = (-b, 0, 0)^T \text{ and } \mathbf{r}_{CM,B} = \left( \frac{m}{M} b, 0, 0 \right)^T
\]

and the normal vector \( \hat{n} \) is now

\[
\hat{n} = (0, 0, 1)^T.
\]

The position of the dampermass with respect to the body frame can therefore be found as

\[
\mathbf{r}_d = (-b, 0, u)^T. \quad (3.83)
\]

For this setup the linearized equations of motion are of course different from the previous case with the horizontal damper. Since the derivation follows the same steps as for the previous one, all intermediate steps are omitted and only the results are given:

\[
\begin{align*}
I_1(\ddot{\alpha}_1 - \dot{\alpha}_2 \nu) + (I_3 - I_2)(\ddot{\alpha}_2 \nu - \alpha_1 \nu) &= 0, \\
I_2(\ddot{\alpha}_2 + \dot{\alpha}_1 \nu) + (I_1 - I_3)(\ddot{\alpha}_1 - \alpha_2 \nu) + mb\ddot{u} + mbu^2 \dot{u} &= 0, \\
I_3\ddot{\alpha}_3 &= 0, \\
mb\ddot{u} + d\dot{u} + cu + mb(\ddot{\alpha}_2 + \alpha_2 \nu^2) &= -mg \quad (3.84)
\end{align*}
\]

Written in matrix form and omitting the equation for \( \alpha_3 \) we yield:

\[
\mathbf{M} \cdot \ddot{\mathbf{q}} + (\mathbf{G} + \mathbf{D}) \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = \mathbf{F} \quad (3.85)
\]
with the following definitions for the matrices:

\[
M = \begin{bmatrix}
I_1 & 0 & 0 \\
0 & I_2 & mb \\
0 & mb & m
\end{bmatrix},
\]  
(3.86)

\[
G = (I_3 - I_2 - I_1)\nu \begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]  
(3.87)

\[
D = diag\{0, 0, 0, d\},
\]  
(3.88)

\[
K = \begin{bmatrix}
(I_3 - I_2)\nu^2 & 0 & 0 \\
0 & (I_3 - I_1)\nu^2 & mb\nu^2 \\
0 & mb\nu^2 & c
\end{bmatrix},
\]  
(3.89)

\[
F = diag\{0, 0, -mg\}.
\]  
(3.90)

The equations agree with the equations derived in Ref [5]. Surprisingly the signs of the \(mb\)-terms in the \(M\)-Matrix and the \(mb\nu^2\)-terms in the \(K\)-matrix are the same in both derivations, though the damper is placed on the negative axis in our case. The reason for this is that since Hughes’s coordinate system has a different orientation than the one used in this report, the definition for the linearized angular velocity vector differs, which causes the unexpected coincidence of the signs. To doublecheck the equations were linearized using the same damper orientation as Hughes which lead to agreeing results.
4 Setup of Simulation

The MATLAB simulation consists of three main scripts: `iohandler.m`, `config.m` and `equofmo.m`. Each individual script is explained in the following chapter and the code is attached in the appendix. The principal setup can be seen in figure (4.5).

![Flowchart of simulation](image)

**Figure 4.5: Flowchart of simulation**

4.1 Description of `config.m`

The script `config.m` contains all initial conditions and parameters to fully describe the system. At the beginning of the simulation this script is read out and all values are stored to the MATLAB workspace. The advantage of having all variables stored in one script is to have a better overview over the complex system. Furthermore different system configurations can be saved in different configuration files. This way, multiple simulations can easily be reproduced and compared. The initial conditions and parameters to be determined to describe the system are listed and explained in table (4.1). Note that all vectors must be expressed in the body frame.

Since the equations of motion are written in terms of the angular momentum vector and the linear momentum of the damper mass, those two values must be calculated and they are used as initial conditions for the integration. The initial kinetic energy must
<table>
<thead>
<tr>
<th>Description</th>
<th>Character in code</th>
<th>Format</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Global parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gravity field vector</td>
<td>g</td>
<td>3x1 vector</td>
<td>m/s²</td>
</tr>
<tr>
<td>integration interval</td>
<td>tspan</td>
<td>1x2 vector</td>
<td>sec</td>
</tr>
<tr>
<td><strong>Rigid body</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mass of simulator</td>
<td>M</td>
<td>scalar</td>
<td>kg</td>
</tr>
<tr>
<td>Inertia matrix of simulator</td>
<td>L_sim</td>
<td>3x3 matrix</td>
<td>kgm²</td>
</tr>
<tr>
<td>initial attitude in quaternions</td>
<td>q</td>
<td>4x1 vector</td>
<td>1</td>
</tr>
<tr>
<td>initial angular velocity vector</td>
<td>omega</td>
<td>3x1 vector</td>
<td>rad/sec</td>
</tr>
<tr>
<td>position of center of mass</td>
<td>r_cm</td>
<td>3x1 vector</td>
<td>m</td>
</tr>
<tr>
<td><strong>Damper</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>damper mass</td>
<td>m</td>
<td>scalar</td>
<td>kg</td>
</tr>
<tr>
<td>spring stiffness</td>
<td>c</td>
<td>scalar</td>
<td>N/m</td>
</tr>
<tr>
<td>damping coefficient</td>
<td>d</td>
<td>scalar</td>
<td>N·sec/m</td>
</tr>
<tr>
<td>orientation of damper</td>
<td>n</td>
<td>3x1 vector</td>
<td>1</td>
</tr>
<tr>
<td>massposition for relaxed spring</td>
<td>r_d0</td>
<td>3x1 vector</td>
<td>m</td>
</tr>
<tr>
<td>initial deflection from relaxed pos.</td>
<td>u0</td>
<td>scalar</td>
<td>m</td>
</tr>
<tr>
<td>initial velocity of mass</td>
<td>v0</td>
<td>scalar</td>
<td>m/sec</td>
</tr>
<tr>
<td><strong>Drag modell</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>density of air</td>
<td>rho_air</td>
<td>scalar</td>
<td>kg/m³</td>
</tr>
<tr>
<td>radius of simulator</td>
<td>r_sim</td>
<td>scalar</td>
<td>m</td>
</tr>
<tr>
<td>height of superstructure</td>
<td>height</td>
<td>scalar</td>
<td>m</td>
</tr>
<tr>
<td>drag coefficient</td>
<td>C_d</td>
<td>scalar</td>
<td>1</td>
</tr>
<tr>
<td><strong>Calculated values</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>initial angular momentum vector</td>
<td>H_0</td>
<td>3x1 vector</td>
<td>kgm²/sec</td>
</tr>
<tr>
<td>initial linear momentum</td>
<td>p_d_0</td>
<td>scalar</td>
<td>kgm/sec</td>
</tr>
<tr>
<td>initial kinetic energy</td>
<td>T_0</td>
<td>scalar</td>
<td>J</td>
</tr>
</tbody>
</table>

Table 4.1: Overview over simulation parameters
also be included in the initial conditions since it was incorporated into the equations of motion as a control value.

4.2 Description of iohandler.m

The core part of the simulation is the input-output-handle script iohandler.m. After the integration is started the configuration script as described in the last chapter is read in. Next the integration is initialized with the initial conditions and all necessary parameters. The built in ODE-solvers from MATLAB were used to integrate the equations of motion. After the integration is complete, the solution is obtained in terms of quaternions, angular momentum vector, linear momentum and position of the damper mass and kinetic energy. In order to have the angular momentum vector and the velocity of the mass explicitly, these values must be extracted from the angular momentum and the linear momentum. Equation (3.21) and equation (3.12) can be rewritten in matrix form as:

\[
\begin{bmatrix}
H \\
p_{d,n}
\end{bmatrix} = \begin{bmatrix}
I & m(r_d \times \hat{n}) \\
-m\hat{n}^T r_d & m
\end{bmatrix} \cdot \begin{bmatrix}
\omega \\
\dot{u}
\end{bmatrix}. \tag{4.91}
\]

The time-varying intetria matrix \(I\) of the whole system can be easily calculated with the known position of the damper mass at each timestep. Therefore the set of linear equations in equation (4.91) can be solved for the angular velocity vector and the velocity of the mass. This happens in a loop over the total integration time.

In the same loop the kinetic energy can be calculated according to equation (3.42). Now the value for kinetic energy obtained from the solution for \(\omega\) and \(\dot{u}\) can be compared to the integrated kinetic energy rate. The values should be equal at all times and the magnitude of their difference is a measure for the quality of the integration. It was observed that the solver ODE45 yields poor results if used with the default setting for error tolerance. In order to achieve satisfying accuracy in a reasonable amount of time an absolute and relative error tolerance of \(10^{-6}\) was used. After the integration is finished and all necessary values are computed in the loop the results are returned to the workspace for further preprocessing. For this purpose scripts which create different useful plots were written and are attached in the appendix but not further discussed in this chapter.

4.3 Description of equofmo.m

In the script equofmo.m the equations of motion for the total system are set up for the numerical integration. The initial conditions needed to integrate the equations of motion are the initial quaternion vector \(\bar{q}_0\), the initial angular momentum vector \(H_0\), the initial linear momentum vector \(p_{d,n}\), the initial deviation of the damper mass \(u\) and the initial kinetic energy \(T_0\). The MATLAB ODE-solver requires a set of first order differential equations. We recall the equations of motion and the kinematic equation
and set $\dot{u} = v$, which introduces an additional equation:

$$
\dot{\bar{q}} = \frac{1}{2} \left[ \mathbf{q}^\times + \mathbf{q}_d \mathbf{E} \right] \cdot \omega,
$$

(4.92)

$$
\dot{p}_{d,n} = \hat{n} \cdot (-\omega \times p_d) + \hat{n} \cdot F_{g,P} - cu - dv,
$$

(4.93)

$$
\dot{H} = -\omega \times H + r_{cm} \times F_{g,B} + r_d \times F_{g,P} + r_{th} \times F_{Th},
$$

(4.94)

$$
\dot{u} = v,
$$

(4.95)

$$
\dot{T} = \omega \cdot (r_{cm} \times F_{g,B} + r_d \times F_{g,P} + r_{th} \times F_{Th}) + \ddot{u}(-cu - dv) + \dot{u}F_{g,P} \cdot \hat{n}.
$$

(4.96)

Note that the derivatives on the left hand side of these equations are all with respect to the body-fixed frame. The values for $p_{d,n}$ and $H$ can be easily calculated from

$$
p_{d,n} = m(v - m\hat{n}^T r_d \hat{\omega}),
$$

(4.97)

$$
H = I\omega + mv(r_d \times \hat{n})
$$

(4.98)

while $v$ and $\omega$ must be calculated from equation (4.91) previous to every integration step.
5 Stability Analysis of the Linearized Equations of Motion

In order to develop design criteria for the damping mechanism it is necessary to examine the stability of the dynamical system. There exist a large number of definitions for stability with different advantages and disadvantages if applied to analyze a specific system. An overview of the most important theories is given in appendix A of Ref. [5]. The most general definition useful for mechanical systems is provided by Liapunov. He stated that a system is stable, if the solution for the perturbed system never exceeds a certain bound around the unperturbed solution. If this statement is true, the system is said to be L-stable.

Note that it is not required in the previous definition, that the system returns to its nominal configuration after the perturbation was applied. Therefore it is helpful to establish the definition of attractive stability. In this case the system returns to its nominal state for \( t \to \infty \) if the perturbation is sufficiently small.

Finally taking both definitions together leads to the definition of asymptotic stability:

* A system is asymptotically stable if its solution is L-stable and attractive.

This definition is much more suited to real mechanical systems than the simple Liapunov stability, since the system might undergo a series of perturbations. Though the effects of their individual disturbances might be limited to a certain bound, their summation can still exceed a maximum acceptable bound. A system which is asymptotically stable according to the definition above, always returns to its nominal state previous to the perturbation.

Let us consider stability of a simple damped linear oscillator with the equation of motion:

\[
\ddot{x} + 2D\omega_0\dot{x} + \omega_0^2x = 0 \tag{5.99}
\]

In this equation \( D \) is the damping ratio and \( \omega_0 \) is the Eigenfrequency of the undamped system. The solution to this equation is well known and can be found as:

\[
x(t) = e^{\sigma t} \cdot (A \cos(\omega t) + B \sin(\omega t)) \tag{5.100}
\]

with \( \sigma = -\omega_0 D \) and the frequency of the damped system \( \omega = \omega_0 \sqrt{1 - D^2} \). The constants \( A \) and \( B \) need to be determined from the initial conditions.

It becomes obvious from these equations that Liapunov stability is given for \( \sigma = 0 \), because for an initial deviation and velocity (i.e. a perturbation) the system will oscillate around the stationary solution. In case damping is present the value for \( \sigma \) becomes negative and the system is asymptotically stable. The system is unstable for negative damping, because the exponential function grows larger and larger and the motion exceeds all limits.

The characteristic equation for (5.99) can be found by substituting \( x = e^{\lambda t} \):

\[
\lambda^2 + 2D\omega_0\lambda + \omega_0^2 = 0. \tag{5.101}
\]
This quadratic equation can be solved for \( \lambda \):

\[
\lambda_{1,2} = -D\omega_0 \pm \omega_0 \sqrt{D^2 - 1} = \sigma \pm i\omega, \tag{5.102}
\]

with \( i = \sqrt{-1} \). \( \lambda_{1,2} \) are called the roots of the characteristic equation and as seen above, an examination of the roots above gives a quick insight into the dynamical behaviour of the system without having to calculate an analytical solution to it. If plotted in the \( \sigma - i\omega \)-plane, we find that asymptotical stability is given if the roots of the characteristic equations lie in the left hand side. The system is unstable is any roots accur in the right hand plane.

A quick and efficient way to examine stability of a dynamical system is provided by the Routh-Hurwitz-criterium. It states that if the characteristic equation is given in the form

\[
p(\lambda) = a_0\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n \tag{5.103}
\]

the system is unstable in case one of the \( a_i < 0 \). This is a necessary condition for stability. To derive the sufficient conditions of asymptotic stability, the so called Hurwitz matrix must be studied. This matrix is formed by aligning the coefficients with uneven and even indices in two rows, which form the first two rows of the matrix. The next rows are found by adding zeros to the beginning until the coefficient with the highest index reaches the lower right corner of the matrix:

\[
H = \begin{bmatrix}
  a_1 & a_3 & a_5 & \ldots & 0 \\
  a_0 & a_2 & a_4 & \ldots & 0 \\
  0 & a_1 & a_3 & \ldots & 0 \\
  0 & a_0 & a_2 & \ldots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & 0 & \ldots & a_n
\end{bmatrix} \tag{5.104}
\]

The sufficient conditions for stability are fulfilled for the following determinants being greater than zero:

\[
\Delta_1 = a_1 > 0 \\
\Delta_2 = \det \begin{bmatrix}
  a_1 & a_3 \\
  a_0 & a_2
\end{bmatrix} > 0 \\
\Delta_3 = \det \begin{bmatrix}
  a_1 & a_3 & a_5 \\
  a_0 & a_2 & a_4 \\
  0 & a_1 & a_3
\end{bmatrix} > 0 \\
\vdots \\
\Delta_n = \det H > 0
\]

With the help of the necessary conditions these conditions can be further simplified. The necessary conditions are listed in Ref [5], page 497, Table A.1. For later use, the conditions for a sixth-order system are here recalled:

\[
a_1, a_2, a_4, a_6, \Delta_3, \Delta_5 > 0, \tag{5.111}
\]
or equivalently
\[ a_1, a_3, a_5, a_6, \Delta_3, \Delta_5 > 0. \] (5.112)

## 5.1 Horizontal configuration

In order to analyze the stability of the damper in horizontal configuration, the characteristic equation of the system must be obtained. Setting \( q_i = \dot{q}_i e^{st} \) in equation (3.78) we find

\[
M \cdot s^2 + (G + D)s + K = 0. \quad (5.113)
\]

The characteristic equation is now the determinant of the equation above:

\[
\det \begin{bmatrix}
I_1s^2 + (I_3 - I_2)\nu^2 & (I_3 - I_1 - I_2)\nu & 0 & gm \\
-(I_3 - I_1 - I_2)\nu & I_2s^2 + (I_3 - I_1)\nu^2 & 0 & 0 \\
0 & 0 & I_3s^2 & -bms^2 \\
0 & 0 & -bms^2 & c + ds + ms^2 - m\nu^2
\end{bmatrix} = 0. \quad (5.114)
\]

The characteristic equation can now be calculated and is found to be of the form:

\[
s^2 \sum_{n=0}^{6} a_n s^{n-6} = 0. \quad (5.115)
\]

The coefficients \( a_n \) are:

\[
\begin{align*}
  a_0 &= I_1I_2m(I_3 - mb^2) \\
  a_1 &= (dI_1I_2I_3) \\
  a_2 &= cI_1I_2I_3 + m((I_3 - I_2)I_3(I_3 - mb^2) + I_1(I_2I_3 - I_3^2 - 2b^2I_2m + b^2I_3m)\nu^2 \\
  a_3 &= dI_3(2I_1I_2 - I_1I_3 - I_2I_3 + I_3^2)\nu^2 \\
  a_4 &= -g^2I_2I_3m^2 - \nu^2(cI_3((I_2 - I_3)I_3 + I_1(I_3 - 2I_2)) + m(mb^2I_3(I_3 - I_2) + I_1(I_2I_3 + mb^2(I_2 - I_3))\nu^2) \\
  a_5 &= d(I_3 - I_1)(I_3 - I_2)I_3\nu^2 \\
  a_6 &= (I_1 - I_3)I_3\nu^2(g^2m^2 + (I_2 - I_3)\nu^2(c - \nu^2))
\end{align*}
\]

As already discussed above, the coefficients and subdeterminants must be checked for their signs. The constants which appear in the equations are the inertias \( I_1, I_2 \) and \( I_3 \), the mass \( m \), the spring constant \( c \), the damping ratio \( d \), the gravitational acceleration \( g \) and the nominal spin speed \( \nu \). All of these values are defined to be positive. The coefficient \( a_0 \) includes the term \((I_3 - mb^2)\). We can write \( I_3 \) according to equation (3.69) as \( I_3 = I_{3\text{sim}} + mb^2 \), where \( I_{3\text{sim}} \) is the moment of inertia of only the rigid body about the z-axis which is always positive. Therefore the coefficient \( a_0 \) is always positive. It is trivial to see that the coefficient \( a_1 \) is also always positive. The coefficient \( a_5 \) is positive for:

\[ I_3 > I_1 \text{ and } I_3 > I_2 \quad \text{or} \quad I_3 > I_1 \text{ and } I_3 > I_2. \]

This condition can easily be recognized as the major or minor axis rule. It states that stability for a spinning rigid body is only given, if it spins around its major or
minor axis. In our case the $\hat{b}_3$-axis is the nominal spin axis and it must be the major or minor axis, according to this condition. As we will see later this condition needs to be restricted to only the major axis.

The sign of the coefficient for $a_6$ is not obvious at the first glance, but it can be rewritten as the inequality $a_6 > 0$ and solved for a minimum spring stiffness $c$:

$$c_{\text{min}} > \frac{(mg)^2 + (I_3 - I_2)\nu^4}{(I_3 - I_2)\nu^2}.$$  

(5.116)

This equation means that for a given damper mass, given inertias $I_2$ and $I_3$ and a spin speed the spring needs to have a certain minimal stiffness to make the mass return to its nominal position. Otherwise the system becomes unstable.

Now the subdeterminants $\Delta_3$ and $\Delta_5$ need to be examined:

$$\Delta_3 = (dI_1 I_2)^2 I_3^3 (2I_1 I_2 - I_1 I_3 - I_2 I_3 + I_3^2)\nu^2 (c - m\mu^2),$$  

(5.117)

$$\Delta_5 = d^3 g^4 I_1^2 I_3^2 (I_3 - I_1)(I_1 + I_3 - I_3) 2m^4 \nu^4.$$  

(5.118)

The only two terms in the equation for $\Delta_3$ which can switch their signs are the two parantheses. We recognize that the first paranthesis is exactly coefficient $a_3$, which was not yet examined since it is the most difficult one. Since $a_3$ needs to be positive, the expression in the second paranthesis must also be positive at all times. This is already ensured in equation (5.116), since the stability condition for the spring stiffness without gravity is $c_{\text{min}} > m\nu^2$.

In order to make $\Delta_5 > 0$ we need to assure that $(I_3 - I_1) > 0$ which means that $I_3$ has to be the major axis. This is an expected result since a gyroscopic system with energy dissipation is only stable if it spins about its major axis, which corresponds to the minimal kinetic energy (Ref [10], Chapter 4.3.1).

The last coefficient $a_3$ is not as obvious as the previous ones. The expression in the paranthesis needs to be greater than zero to make the coefficient positive. We can find the inequality:

$$(2I_1 I_2 - I_3 (I_1 + I_2) + I_3^2) > 0.$$  

(5.119)

We already found that $I_3$ has to be the major axis. Therefore we rewrite inequality (5.119) in terms of the inertia ratios $A = \frac{I_1}{I_3}$ and $B = \frac{I_2}{I_3}$ with $0 < A < 1$ and $0 < B < 1$:

$$(2AB - (A + B) + 1) > 0.$$  

(5.120)

If we define:

$$f = (2AB - (A + B) + 1) = (2B - 1)A - B + 1, \quad f > 0$$  

(5.121)

we see that $f$ is a set of straight lines depending on the parameter $B$. If $f$ is plotted over $A$ in the interval $[0, 1]$ and $B$ is varied in the same interval, it can be observed that $f$ never violates its constraint and stays positive in this interval (see plot in figure(5.6)).
In other words, as long as $I_3$ is the major axis the coefficient $a_3$ stays positive for all possible values of $I_1$ and $I_2$.

In summary the system of the rigid body with the attached spring mass damper is only stable if the following two conditions apply:

1. The spin axis must be the major axis.
2. The spring stiffness must satisfy $c_{min} > \frac{(mg)^2 + (I_3-I_2)m\nu^4}{(I_3-I_2)\nu^2}$.

### 5.2 Vertical Configuration

Now the stability conditions for the vertical configuration are derived. The derivation is very similar to the one provided in Ref. [5] but in this case the linearized equations of motion are left dimensional. Furthermore since our system is under the influence of gravity, a term with the gravity force acting on the damper mass appears on the right hand side of equation (3.85). In order to do the stability analysis in the same way as for the vertical system it would be convenient if this term vanished.
Let us consider a one-dimensional spring-mass system under influence of gravity. the spring is relaxed for $x = 0$. The equation of motion is given by:

$$\ddot{x} + \frac{d}{m} \dot{x} + \frac{c}{m} x = -g,$$

which is a second order inhomogeneous differential equation. After all motion is damped out of the system for $t \to \infty$ the mass will remain in its stationary solution found by setting $\ddot{x}$ and $\dot{x}$ in (5.122) equal to zero:

$$x_\infty = -\frac{mg}{c}.$$

The motion of $m$ can therefore also be described as a damped oscillation around the stationary solution $x_\infty$. The equations of motion can then be rewritten and simplified if we define the spring to be free of forces for $\tilde{x} = -x_\infty$. In this case the spring force is no longer zero for $\tilde{x} = 0$ since it is already elongated by the amount $\|x_\infty\|$. This leads to

$$\ddot{\tilde{x}} + \frac{d}{m} \dot{\tilde{x}} + \frac{c}{m} \tilde{x} = -g,$$

$$\ddot{\tilde{x}} + \frac{d}{m} \dot{\tilde{x}} + \frac{c}{m} \tilde{x} = 0.$$

In equation (5.124) $\tilde{x}$ now describes an excursion from the stationary solution and the inhomogeneity drops out.

The same transformation can be done for the linearized equations for the vertical damper. If we define the spring to be relaxed for $u = -\frac{mg}{c}$ and substitute this into the equations (3.84) we yield:

$$I_1(\ddot{\alpha}_1 - \dot{\alpha}_2 \nu) + (I_3 - I_2)(\ddot{\alpha}_2 - \alpha_1 \nu) = 0,$$

$$I_2(\ddot{\alpha}_2 + \dot{\alpha}_1 \nu) + (I_2 - I_3)(\alpha_1 - \alpha_2 \nu) + mb\ddot{u} + mbv^2 u = 0,$$

$$m\ddot{u} + d\dot{u} + cu + mb(\ddot{\alpha}_2 + \alpha_2 \nu^2) = 0.$$

The matrix form of equations (5.125) is now free of the inhomogeneity:

$$M \cdot \ddot{q} + (G + D) \dot{q} + K \cdot q = 0.$$

Again we substitute $q_i = \tilde{q}_i e^{\nu t}$ into equation (5.126) and we find

$$M \cdot s^2 + (G + D)s + K = 0.$$

The characteristic equation is then the determinant of the equation above:

$$\det \begin{bmatrix} I_1 s^2 + (I_3 - I_2)\nu^2 & (I_3 - I_1 - I_2)\nu s & 0 \\ -(I_3 - I_1 - I_2)\nu s & I_2 s^2 + (I_3 - I_1)\nu^2 & mbv^2 + mbs^2 \\ 0 & mbv^2 + mbs^2 & c + ds + ms^2 \end{bmatrix} = 0,$$

and it is of the form:

$$(s^2 + \nu^2) \sum_{n=0}^{4} a_n s^{n-4} = 0.$$
The term \((s^2 + \nu^2)\) shows that two of the roots of the characteristic equation lie on the imaginary axis. The position of the other roots in the \(\sigma-j\omega\)-plane determines the stability of the system. Again the Hurwitz criteria are used for the analysis. The coefficients \(a_n\) are:

\[
\begin{align*}
a_0 &= (I_1 m(I_2 - mb^2)) \\
a_1 &= dI_1 I_2 \\
a_2 &= cl_1 I_2 I_3 + (m(I_1(I_2 - I_3 - b^2m) - (I_2 - I_3)(I_3 - b^2m))\nu^2 \\
a_3 &= d(I_1 - I_3)(I_2 - I_3)\nu^2 \\
a_4 &= (I_2 - I_3)\nu^2(c(I_1 - I_3) + b^22m^2\nu^2)
\end{align*}
\]

For a fourth order system we need to show that \(a_0, a_1, a_3, a_4, \Delta_3\) are positive. The coefficient \(a_0\) is always positive since \(I_2\) is defined as \(I_2 = I_{2,\text{sim}} + mb^2\). The proof for \(a_1 > 0\) is trivial and \(a_3\) turns out to correspond to the major and minor axis rule and is positive for:

\[
I_3 > I_1 \text{ and } I_3 > I_2 \quad \text{ or } \quad I_3 < I_1 \text{ and } I_3 < I_2.
\]

The coefficient \(a_4\) can be written as the inequality \(a_3 = (I_2 - I_3)\nu^2(c(I_1 - I_3) + b2m^2\nu^2) > 0\) and can be solved for the spring stiffness \(c\):

\[
c_{\min} > \frac{b^2m^2\nu^2}{I_3 - I_1}.
\] (5.130)

As expected we also find a minimum condition for the spring stiffness like we did in the previous case. The analysis of the subdeterminant is left over. \(\Delta_3\) turns out to be:

\[
\Delta_3 = b^2d^2I_1(I_3 - I_2)(I_1 + I_2 - I_3)^2I_3 m^2\nu^4 > 0.
\] (5.131)

We conclude that \(I_3 > I_2\) to make the expression positive and together with the results of the coefficient \(a_3\) we find again that \(I_3\) has to be the major axis.

In summary the stability criteria are:

1. The spin axis must be the major axis.
2. The minimum spring stiffness is given by \(c_{\min} > \frac{m^2b^2\nu^2}{I_3 - I_1}\).

It is important to notice that the stability criteria derived in this and the previous chapter only guarantee that the system is stable, i.e. that it returns to its nominal state which is the simple spin around the \(\vec{b}_3\)-axis for sufficiently small perturbations. In order to make the damper most effective different conditions for the design parameter must be found. In the next chapter a brief overview over precession and nutation of a rigid body is given and conditions under which the damper works optimally are later deduced.
6 Precession and Nutation of Rigid Bodies

In this chapter the equations describing the precession and nutation of a rigid body are reviewed and the difference between axisymmetric and non-axisymmetric bodies is shown. These results are later useful to find the parameters which influence the optimal working point of the damper.

First a description of the relative orientation between the body-fixed frame \( \{b\} \) and the inertial frame \( \{n\} \) must be found. For gyroscopic systems it is convenient to make use of the (3-1-3)-EULER angle sequence with the angles \( \Psi, \theta \) and \( \phi \). These three specific angles are called precession (\( \Psi \)), nutation (\( \theta \)) and spin (\( \phi \)).

The direction cosine matrix for the 3-1-3-rotation can be found by multiplying equations (A.204) and (A.202) from appendix A together in the correct order:

\[
R_{bn} = R_3(\phi) \cdot R_1(\theta) \cdot R_3(\Psi)
\]

\[
= \begin{bmatrix}
  c(\phi)c(\Psi) - c(\theta)s(\phi)s(\Psi) & c(\theta)c(\Psi)s(\phi) + c(\phi)s(\Psi) & s(\theta)s(\phi) \\
  -c(\Psi)s(\phi) - c(\theta)c(\phi)s(\Psi) & c(\theta)c(\phi)c(\Psi) - s(\phi)s(\Psi) & c(\phi)s(\theta) \\
  s(\theta)s(\Psi) & -c(\Psi)s(\theta) & c(\theta)
\end{bmatrix}
\]

Note that \( c \) stands for \( \cos \) and \( s \) for \( \sin \). The next step is to write the angular velocity vector expressed in the body-fixed frame:

\[
\omega = \omega_1 \mathbf{b}_1 + \omega_2 \mathbf{b}_2 + \omega_3 \mathbf{b}_3.
\]

Note that the components of the angular velocity vector are not equal to the EULER angle rates, since the latter are defined around different intermediate axes. To express \( \omega \) in terms of EULER angles, the three rates \( \dot{\Psi}, \dot{\theta} \) and \( \dot{\phi} \) need to be added with respect to the rotation of their associated axes:

\[
\omega = R_3(\phi) \cdot R_1(\theta) \cdot R_3(\Psi) \cdot \dot{\Psi} \mathbf{n}_3 + R_3(\phi) \cdot \dot{\phi} \mathbf{b}_3.
\]

Since the first rotation is around the 3-axis of the inertial coordinate frame, the rate-vector needs to be transferred into the body-frame by all three rotations. The second rotation is around the intermediate 1-axis (denoted as \( \mathbf{b}_1' \)), therefore the rate-vector only needs to be multiplied with the last direction cosine matrix, since the second rotation does not affect the intermediate 1-axis. The third rotation is done around the body-fixed 3-axis, therefore the rate vector is already the \( \{b\} \)-frame.

Written in matrix form equation (6.134) yields the following result:

\[
\omega = \begin{bmatrix}
  \sin(\phi)\sin(\theta) & \cos(\phi) & 0 \\
  \sin(\theta)\cos(\phi) & -\sin(\phi) & 0 \\
  \cos(\theta) & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
  \dot{\Psi} \\
  \dot{\theta} \\
  \dot{\phi}
\end{bmatrix}.
\]
To express the Euler angle rates in terms of $\omega$, the matrix in equation (6.135) needs to be inverted. This leads to the following equation:

\[
\begin{pmatrix}
\dot{\Psi} \\
\dot{\theta} \\
\dot{\phi}
\end{pmatrix} = \frac{1}{\sin(\theta)} \cdot \begin{pmatrix}
\sin(\phi) & \cos(\phi) & 0 \\
\sin(\theta)\cos(\phi) & -\sin(\theta)\cos(\phi) & 0 \\
-\cos(\theta)\sin(\phi) & -\cos(\theta)\sin(\phi) & \sin(\theta)
\end{pmatrix} \cdot \omega.
\] (6.136)

It becomes obvious that equation (6.136) for this symmetric rotation sequence contains a singularity for $\theta = 0^\circ$ or $\theta = 180^\circ$. In that case precession and spin can not be separated and individually determined after the rotation. For an asymmetric rotation sequence (for example a 1-2-3-sequence) this problem occurs if the middle angle equals $90^\circ$ or $270^\circ$.

We recall the definition of the angular momentum vector for a rigid body over its center of mass:

\[
\mathbf{H} = \mathbf{I}_b \cdot \omega.
\] (6.137)

The matrix $\mathbf{I}_b$ is the inertia matrix calculated in the \{b\}-frame and taken over the center of mass.

According to EULER’s rotational equation the inertial derivative of the angular momentum vector is equal to the applied external torques:

\[
\left( \frac{d\mathbf{H}}{dt} \right)^N = \mathbf{M}.
\] (6.138)

It becomes obvious from this equation that if no external torques are applied to the system the angular momentum vector stays constant in orientation and magnitude in the inertial frame. With equations (6.137) and (6.138) it is only a small step to EULER’s rotational equations of motion. If the derivative of equation (6.137) is taken according to the transport theorem we find:

\[
\left( \frac{d\mathbf{H}}{dt} \right)^N = \left( \frac{d}{dt} \right)^N (\mathbf{I}_b \cdot \omega) = \mathbf{I} \cdot \dot{\omega} + \omega \times \mathbf{I}_b \cdot \omega = \mathbf{M}
\] (6.139)

Regrouping this equation yields:

\[
\dot{\omega} = \mathbf{I}^{-1} \cdot ( -\omega \times \mathbf{I} \cdot \omega + \mathbf{M}).
\] (6.140)

Equation (6.140) finally describes the dynamics of a rotating rigid body. In order to achieve analytical solutions to this equation it is necessary to restrict the geometry of the body to certain cases under loss of generality. If the body-fixed frame is aligned with the principal axes the inertia matrix becomes diagonal and we can split (6.140) up into three equations:

\[
\begin{align*}
\dot{\omega}_1 &= \frac{I_2 - I_3}{I_1} \omega_3 \cdot \omega_2 + \frac{M_1}{I_1}, \\
\dot{\omega}_2 &= \frac{I_3 - I_1}{I_2} \omega_3 \cdot \omega_1 + \frac{M_2}{I_2}, \\
\dot{\omega}_3 &= \frac{I_3 - I_2}{I_3} \omega_2 \cdot \omega_1 + \frac{M_3}{I_3}.
\end{align*}
\] (6.141)

In the following sections equations (6.141) to (6.143) are solved for different geometries, namely for an axisymmetric body and a non-axisymmetric body.
6.1 Axisymmetric Body

For the special case of an axisymmetric body the inertia matrix can be further simplified to $I_1 = I_2 = I_t$ and $I_3 = I_s$. According to our application the $I_3$-axis is chosen as the symmetric spin axis and the inertia matrix is diagonal $I = \text{diag}\{I_t, I_t, I_s\}$.

Putting the inertia matrix back into equation (6.141) to (6.143) yields:

\begin{align*}
\dot{\omega}_1 &= -\frac{I_s - I_t}{I_t} \omega_3 \cdot \omega_2 + \frac{M_1}{I_t}, \\ 
\dot{\omega}_2 &= \frac{I_s - I_t}{I_t} \omega_3 \cdot \omega_1 + \frac{M_2}{I_t}, \\ 
\dot{\omega}_3 &= \frac{M_3}{I_s}.
\end{align*}

(6.144) (6.145) (6.146)

These equations are a set of coupled linear first order differential equations for $\omega$. It becomes obvious that, in the case of an axisymmetric body, the angular velocity around the rotation axis stays constant for the torque-free case. Therefore the term $\frac{I_s - I_t}{I_t} \omega_3$ is also constant. As mentioned above, the first two equations are coupled, which means that an angular velocity around the 2-axis leads to an acceleration around the 1-axis and vice versa. The solution for the torque-free case is well known and can be found in Ref. [10], chapter 4.2.3:

\begin{align*}
\omega_1 &= \omega_{1,0} \cos(\omega_p t) - \omega_{2,0} \sin(\omega_p t), \\ 
\omega_2 &= \omega_{2,0} \cos(\omega_p t) + \omega_{1,0} \sin(\omega_p t), \\ 
\omega_3 &= \omega_{3,0},
\end{align*}

(6.147) (6.148) (6.149)

where $\omega_p$ is defined as $\omega_p = \frac{I_s - I_t}{I_t} \omega_{3,0}$.

A plot of the angular velocities over time is shown in figure (6.7). The value for $\omega_3$ stays constant while the angular velocities $\omega_1$ and $\omega_2$ change periodically. The frequency of this periodicity is given by $\omega_p$ and will later become important if the damper needs to be tuned.

Now an analytical expression for the EULER-angle rates needs to be found in order to find a description of the attitude of the body frame. We utilize the expression for the angular momentum vector in equation (6.137):

$$H = I_b \cdot \omega = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \cdot \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}. \quad (6.150)$$

If we now suppose that $H$ is aligned with the inertial 3-axis we can write it in terms of the inertial frame as:

$$^nH = \begin{bmatrix} 0 \\ 0 \\ H \end{bmatrix}. \quad (6.151)$$
If it is transformed into the \(\{b\}\)-frame, we find the following expression using the rotation matrix in equation (6.132):

\[
^{b}H = R_{bn} \cdot ^{n}H = \begin{bmatrix}
0 & 0 & H \\
H \sin(\theta) \sin(\phi) & H \sin(\theta) \cos(\phi) & H \cos(\theta)
\end{bmatrix}.
\] (6.152)

Equation (6.151) and equation (6.152) are combined and solved for \(\omega\):

\[
\omega = \begin{bmatrix}
H \sin(\theta) \sin(\phi) \\
H \sin(\theta) \cos(\phi) \\
H \cos(\theta)
\end{bmatrix}.
\] (6.153)

This result for \(\omega\) is then combined with equation (6.136):

\[
\begin{bmatrix}
\dot{\Psi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} = \begin{bmatrix}
\frac{H \cos(\phi)^2}{I_2} + \frac{H \sin(\phi)^2}{I_1} \\
\frac{H(I_2 - I_1) \sin(\theta) \cos(\phi) \sin(\phi)}{I_1 I_2} \\
\frac{H \cos(\theta)(I_2 I_3 + I_1 (I_3 - 2I_2) + (I_1 - I_2) I_3 \cos(2\phi))}{2 I_1 I_2 I_3}
\end{bmatrix}.
\] (6.154)
Now, the axisymmetric body with $I_1 = I_2 = I_t$ and $I_3 = I_s$ is introduced again and equation (6.154) reduces to a handy expression:

$$\dot{\Psi} = \frac{H}{I_t} \quad (6.155)$$
$$\dot{\theta} = 0 \quad (6.156)$$
$$\dot{\phi} = -\frac{H(I_s - I_t) \cos(\theta)}{I_s I_t} \quad (6.157)$$

These three equations describe the torque-free precession of an axisymmetric body with the Euler angle rates. The angular momentum vector was defined to be aligned with the positive 3-axis and $\Psi$ was defined as a positive rotation around the 3-axis. From equation (6.155) becomes obvious that the bodyfixed 3-axis rotates with a constant rate around the angular momentum vector. This rotation is called precession. According to the chosen definition, $\dot{\Psi}$ is always positive. The nutation rate is according to equation (6.156) equal to zero, therefore the nutation angle $\theta$ between the angular momentum vector and the body fixed rotation axis stays constant. The spin rate $\dot{\phi}$ is also constant for a constant nutation angle. The sign of $\dot{\phi}$ depends on the sign of $\cos(\theta)$ and the ratio of $\frac{I_s - I_t}{I_s I_t}$. For $\theta < 90^\circ$ and a slender body with $I_s < I_t$ precession rate and spinrate have the same sign, which is called direct precession. In the case of a disc shaped body with $I_s > I_t$ the spinrate has a negative sign, which is called retrogade precession.

### 6.2 Asymmetric Body

The analytical solutions for the non-axisymmetric body are not as easily obtainable as for the axisymmetric body. In this case the non-simplyfied equations (6.141) to (6.143) need to be solved. The analytical solution for EULER’s equations in the non-axisymmetric case involves elliptical intergrals and can be found in Ref [11] and [12]. We set $I_3 > I_2 > I_1$ and obtain from the latter reference:

$$\omega_1 = \gamma cn(k, pt), \quad (6.158)$$
$$\omega_2 = \beta sn(k, pt), \quad (6.159)$$
$$\omega_3 = \alpha dn(k, pt). \quad (6.160)$$

The constants $\alpha, \beta, \gamma, k$ and the frequency $p$ are defined as:

$$\alpha = \sqrt{\frac{h^2 - 2I_1 T}{I_3(I_3 - I_1)}}, \quad (6.161)$$
$$\beta = \sqrt{\frac{2I_3 T - h^2}{I_2(I_3 - I_2)}}, \quad (6.162)$$
$$\gamma = -\sqrt{\frac{2I_3 T - h^2}{I_1(I_3 - I_1)}}, \quad (6.163)$$
$$k = \sqrt{\frac{I_2 - I_1}{I_3 - I_2} \frac{2I_3 T - h^2}{h^2 - 2I_1 T}}. \quad (6.164)$$
$p = \sqrt{\frac{(h^2 - 2I_1T)(I_3 - I_2)}{I_1I_2I_3}}. \quad (6.165)$

The parameter $k$ determines the numerical solution for the elliptical functions and $p$ is their frequency. The kinetic energy and the square of the magnitude of the angular momentum vector are defined as:

\begin{align*}
    h^2 &= I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_3^2\omega_3^2, \quad (6.166) \\
    2T &= I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2, \quad (6.167)
\end{align*}

The plot in figure (6.8) shows the three angular rates over time. It can be seen that the angular rate $\omega_3$ of the major axis, which is determined by the function $dn(pt)$, has exactly twice the frequency of the other two rates. This is an important ratio to keep in mind for the later tuning of the damper.

![Plot of angular velocities](image)

**Figure 6.8**: Plot of angular velocities of an asymmetric body

Note that the definition also holds for the axisymmetric case as shown in Ref [11], since in this case $k$ is equal to zero and the elliptic functions reduce to $sn = \sin, cn = \cos, dn = 1$. The frequency $p$ boils down to $p = \frac{I_1-I_2}{I_2}\omega_3$ as already found above.
In order to find an attitude description the matrix-equation (6.154) for the EULER-angles need to be investigated:

\[
\begin{align*}
\dot{\Psi} &= \frac{H \cos(\phi)^2}{I_2} + \frac{H \sin(\phi)^2}{I_1}, \\
\dot{\theta} &= \frac{H(I_2 - I_1) \sin(\theta) \cos(\phi) \sin(\phi)}{I_1 I_2}, \\
\dot{\phi} &= \frac{H \cos(\theta)(I_2 I_3 + I_1(I_3 - 2I_2) + (I_1 - I_2)I_3 \cos(2\phi))}{2I_1 I_2 I_3}.
\end{align*}
\]

(6.168) (6.169) (6.170)

It becomes obvious that the precession rate in equation (6.168) is no longer constant but depends on the spin angle \(\phi\). Furthermore the nutation rate \(\dot{\theta}\) is no longer zero. This leads to a periodic change \(\theta\) called nutation. The spin rate is also a rather complicated term.
7 Tuning of the Damper

In the previous chapters the foundation to find the optimal damper configuration was laid. After we derived the conditions to achieve a stable system, now the conditions to find the most effective damper need to be derived. We call a damper 'tuned' if it dissipates the highest amount of energy in the smallest amount of time, i.e. the highest energy dissipation rate. We know from the derivation of the equations of motion that the energy dissipation rate is found as $\dot{E} = -d\dot{u}^2$ if no external torques are acting. In order to have an effective damper this expression needs to be driven to high values. Therefore the Eigenfrequency of the springmass-damper must be set to the same frequency as the incitation due to the wobble of the simulator. In this case the system is in resonance and the highest values for $u$ can be expected. In the following chapters the conditions for the tuned damper in horizontal and vertical orientation are derived and discussed.

7.1 Vertical Damper

The tuning of a vertical damper is included in Ref. [5], chapter 10 as a textbook problem. The following assumptions are made:

1. The nutation angle is small.
2. The damper mass is small compared to the mass of the simulator.
3. External torques are small and negligible.
4. The damped motion is similar to the undamped motion.

The first assumption allows us to consider the linearized equations (3.84). As already shown previously the inhomogeniety of this equation can be omitted for an appropriate coordinate transformation and the solution for the system can be treated as an oscillation around the stationary solution. We obtain a second-order differential equation with external incitation:

$$\ddot{u} + \frac{d}{m}\dot{u} + \frac{c}{m}u = -b(\ddot{\alpha}_2 + \alpha_2 \nu^2)$$  \hspace{1cm} (7.171)

We omit the expression which inlcudes the EULER angle $\alpha_2$ due to assumption 1 and write $\ddot{\alpha}_2$ as $\dot{\omega}_2$:

$$\ddot{u} + \frac{d}{m}\dot{u} + \frac{c}{m}u = -b\dot{\omega}_2.$$  \hspace{1cm} (7.172)

The Eigenfrequency of this spring mass damper is given by $\omega_0 = \sqrt{\frac{c}{m}}$. in order to achieve resonance, the Eigenfrequency of the system must match the frequency of the oscillatory forcing term. This frequency can be found from the solution for $\omega_2$, which was already found in chapter 6. The axisymmetric case is the easier one and we can obtain the following expression from the derivative of equation (6.148):

$$\dot{\omega}_2 = -\omega_{2,0}\omega_p \sin(\omega_p t) + \omega_{1,0}\omega_p \cos(\omega_p t).$$  \hspace{1cm} (7.173)
The frequency of the incitation is therefore equal to \( \omega_p = \frac{I_s - I_t}{I_t} \omega_{3,0} \). If we set \( \omega_{3,0} = \nu \) and obtain the following condition for the spring-mass ratio to get resonance:

\[
\frac{c}{m} = \left( \frac{I_s - I_t}{I_t} \right)^2 \nu^2, \tag{7.174}
\]

\[
c = m \left( \frac{I_s - I_t}{I_t} \right)^2 \nu^2.
\]

This result correspond to the one found in Ref. [5]. In the case of the non-axisymmetric case the resonance condition is more difficult to find. The solution for \( \omega_2 \) was found in terms of the elliptic function \( sn(pt) \):

\[
\omega_2 = \beta sn(pt). \tag{7.175}
\]

The time derivative of the elliptic function is according to Ref. [12], chapter 13.1 \( \frac{d}{dt} sn(pt) = pcn(pt) dn(pt) \). In Ref. [1] the average values of the elliptical functions are given as:

\[
\begin{align*}
sn(pt) & \approx \sin(pt) \tag{7.176} \\
\cn(pt) & \approx \cos(pt) \\
\dn(pt) & \approx 0.5(1 + \sqrt{1 - k^2})
\end{align*}
\]

Making use of the all these assumptions we finally find:

\[
\dot{\omega}_2 = \beta 0.5(1 + \sqrt{1 - k^2}) \cos(pt). \tag{7.177}
\]

Therefore the damper must be tuned to the frequency \( p \) as calculated in equation (6.165). We find the following value for the spring stiffness:

\[
c = mp^2. \tag{7.178}
\]

The results agree with the results found in in reference [5] and reference [13].

### 7.2 Horizontal Damper

In the same manner as for the vertical damper the resonance frequency and the resulting spring stiffness can be found for the horizontal damper. We consider the linearized equation of motion for the damper mass in equation (3.68):

\[
\ddot{u} - b\dot{\omega}_3 - u\nu^2 + \frac{c}{m} u + \frac{d}{m} \ddot{u} + mg\alpha_1 = 0. \tag{7.179}
\]

Here we assume that the influence of the gravity force is small since the damper is mounted vertically and the nutation angle is small. We omit the term including \( \alpha_1 \) and find a similar second-order differential equation:

\[
\ddot{u} + \frac{d}{m} \ddot{u} + \left( \frac{c}{m} - \nu^2 \right) u = b\dot{\omega}_3 = b\dot{\omega}_3. \tag{7.180}
\]
We now consider the same two symmetry cases as before and start with the axisymmetric case. Since we know from the analysis in chapter 6 that the angular velocity around the symmetry axis remains constant, there is no external incitation visible in equation (7.180). According to the results obtained from the linearized equations the performance of an horizontal damper in an axisymmetric spacecraft or simulator is expected to be very poor. It is shown in reference [1] that the non-linearized equation for the damper mass includes a $\omega_1 \cdot \omega_2$-term. This term falls out during the linearization but for the axisymmetric case its influence becomes significant for large enough nutation angles. The $\omega_1 \cdot \omega_2$-term provides an oscillatory incitation which has twice the precession frequency. Therefore, for large nutation angles, the horizontal damper must be tuned to this frequency:

$$\sqrt{\frac{c}{m} - \nu^2} = 2\rho.$$  \hspace{1cm} (7.181)

The condition for the ideal spring stiffness is then:

$$c = 4\rho^2 m + m\nu^2 = 4\left(\frac{I_s - I_t}{I_t}\right)^2 \nu^2 m + m\nu^2.$$ \hspace{1cm} (7.182)

Now the asymmetric case is considered. The angular velocity $\omega_3$ was already found to be equal to:

$$\omega_3 = \alpha dn(pt).$$ \hspace{1cm} (7.183)

The time derivative of the $dn(pt)$-function is according to Ref. [12] $\frac{d}{dt}dn(pt) = -p^2 cn(pt) sn(pt)$. Using the averaged values for the elliptical functions again we obtain:

$$\dot{\omega}_3 = -\alpha p^2 \sin(pt) \cos(pt) = -\alpha p^2 \sin(2pt).$$ \hspace{1cm} (7.184)

The incitation has again twice the frequency as for the vertical case. Therefore we find the following condition for the spring stiffness:

$$c = 4\rho^2 m + m\nu^2.$$ \hspace{1cm} (7.185)

The frequency $\rho$ is this time calculated according to equation (6.165). For a better overview the equations for the optimal spring stiffness are summarized in table (7.2). Note that the conditions for the spring stiffness in table (7.2) do not necessarily satisfy the stability criteria as previously derived. In particular it must be verified that the influence of gravity for the horizontal damper is indeed small and the system is stable.

<table>
<thead>
<tr>
<th>Damper Type</th>
<th>Axisymmetric Case</th>
<th>Asymmetric Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Damper</td>
<td>$c = m\left(\frac{I_s - I_t}{I_t}\right)^2 \nu^2$</td>
<td>$c = m\left(\frac{h^2 - 2hT_1(T_3 - T_2)}{I_1 I_2 I_3}\right)$</td>
</tr>
<tr>
<td>Horizontal Damper</td>
<td>$c = 4\left(\frac{I_s - I_t}{I_t}\right)^2 \nu^2 m + m\nu^2$</td>
<td>$c = m\left(4\frac{(h^2 - 2hT_1(T_3 - T_2))}{I_1 I_2 I_3} + \nu^2\right)$</td>
</tr>
</tbody>
</table>

Table 7.2: Spring stiffness for tuned dampers
7.3 Verification of the Results

With the help of the simulation we can examine how the damper performance changes if the damper is detuned. The following plots show the effect of the vertical and the horizontal damper. The simulation for an asymmetric body was set up with the same initial conditions each time, only the orientation of the damper was varied. Note that the simulations were performed without the influence of gravity to show that the system is indeed in resonance. The plots in figure (7.9) and figure (7.10) show a vertically installed damper. For the first plot the damper was tuned to be in resonance with the precession of the rigid body. It becomes obvious that the unwanted angular rate around the 1- and 2-axis damp out very well over the simulation time of five minutes. It can also be obtained that the effectiveness of the damper decreases over time, which has two different reasons. First, for smaller nutation angles the amplitude of the incitation as seen from the damper becomes smaller and the damper is less induced to oscillate. Secondly, the damper was tuned according to the initial conditions and therefore to a certain precession frequency. Since the damper dissipates energy, the precession frequency changes and the damper gets more and more detuned and less effective. The huge difference between a tuned and detuned damper can be seen in figure (7.10). The spring is in this case too stiff to create resonance and the damper does not manage to damp out the angular rates in the same period of time.

The two plots in figure (7.11) and figure (7.12) show the influence of a horizontal damper on the motion of the same rigid body. The first plot shows that the tuned damper also reduces the angular rates by a significant amount but the effectiveness is smaller than for the vertical damper. It was in fact shown in references [1] and [6] that the vertical damper is superior to the horizontal damper for sufficiently small nutation angles.

The last plot in figure (7.12) show the less effective damper if the spring stiffness is changed and no resonance occurs. It is clear that the horizontal damper is much more sensitive to detuning than the vertical damper, since only little effect on the angular rates is visible in this case.
Figure 7.9: Plot of angular rates for tuned vertical damper without gravity

Figure 7.10: Plot of angular rates for detuned vertical damper without gravity
Figure 7.11: Plot of angular rates for tuned horizontal damper without gravity

Figure 7.12: Plot of angular rates for detuned horizontal damper without gravity
7.4 Selection of Most Effective Design

In the previous chapters stability criteria and the conditions to tune the damper in order to maximize its effectiveness were developed. Based on these results the feasibility of the different designs is now examined and the most promising configuration is chosen to be realized. According to the plots in figure (7.9) and figure (7.10) one could assume that the vertical damper has a higher effect on the angular rates for small nutation angles. Reference [1] and reference [6] compare the effectiveness, i.e. the energy dissipation rate, of a vertical and a horizontal damper. Note that in this paper the vertical damper is called precession damper and the horizontal damper is referred to as a nutation damper. The moment of inertia of the spin axis is denoted with \( A \), while the axis at which the damper is placed has the moment of inertia denoted by \( C \). The plot in figure (7.13) shows the ratio of the energy dissipation rates for the two configurations.

![Comparison between vertical and horizontal damper](image)

Figure 7.13: Comparison between vertical and horizontal damper, from ref [1]

It becomes obvious that the horizontal damper is superior to the vertical damper if the maximal nutation angle exceeds a certain value. This value depends on the inertia properties of the body. Generalized it can be stated that the vertical damper has a better performance for smaller nutation angles. Due to the configuration of Whorl-I, which only allows for small nutation angles, the vertical damper seems to be preferrable and is checked for feasibility first.
For a first rough estimate if the stability criteria are satisfied some parameters are assumed and shown in table (7.3). The simulator is assumed to be trimmed to principal axes and no external torques are acting on the system. The distance of the damper from the center of rotation corresponds to the radius of the baseplate of Whorl-I. The spin rate was chosen to be 75 °/sec, which is the maximum measurable angular velocity with the rate sensor on Whorl-I. The precession frequency is approximately equal to 1.1 rad/sec.

<table>
<thead>
<tr>
<th>Moments of inertia</th>
<th>$I_1 = 6\text{kgm}^2$; $I_2 = 7\text{kgm}^2$; $I_3 = 12\text{kgm}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damper distance from CR</td>
<td>$b = 0.5\text{ m}$</td>
</tr>
<tr>
<td>Damper mass</td>
<td>$m = 1\text{ kg}$</td>
</tr>
<tr>
<td>Spin rate</td>
<td>$\nu = 75\text{ °/sec} \approx 1.3\text{ rad/sec}$</td>
</tr>
<tr>
<td>Precession frequency</td>
<td>$p =\approx 1.1\text{ rad/sec}$</td>
</tr>
</tbody>
</table>

Table 7.3: Approximate design parameter

If the stability condition in equation (5.130) for the vertical damper is evaluated with these parameters the minimum spring stiffness is equal to $c_{\text{min}} = 0.07\text{N/m}$. For this case the optimal spring stiffness to achieve resonance is according to the equation in table (7.2) equal to $c_{\text{opt}} = 1.21\text{N/m}$ and the system is stable. For a spacecraft this spring mass system could therefore be realized but since the simulator is exposed to the gravitation on earth the elongation of the spring must also be taken into account. For a mass of 1kg and the optimal spring stiffness the elongation would be equal to $u_\infty = \frac{mg}{c_{\text{opt}}} = 5.8\text{m}$. This means that the dampertube needs to be more than six meters long since the spring must be attached 5.8 meters above the simulator’s baseplate. If a smaller mass is chosen the spring stiffness must also be reduced due to constraint $c = mp^2$ and $u_\infty$ remains unchanged. The only practical solution would be to reduce the weight of the mass and make the spring stiffer in order to achieve an acceptable damper length. As a consequence the damper becomes detuned from its optimal configuration and its effectiveness would be reduced. It was shown with the simulation for different combination of spring stiffness and mass that the damping effect becomes negligible. Therefore the vertical configuration turns out not to be realizable.

Next the feasibility of the horizontal damper is checked. If the same parameters are assumed as for the previous case we find the following minimum spring stiffness from equation (5.116): $c_{\text{min}} = 13\text{N/m}$. The optimal spring stiffness can be found again from the equations in (7.2) to be equal to $c_{\text{opt}} = 6.55\text{N/m}$ and the system would be unstable in this case. The optimal and minimal spring stiffness are plotted over different values for the damper mass in figure (7.14) for the spin rate of 75°/sec. It becomes obvious that for smaller masses the optimal spring stiffness becomes larger than the minimal spring stiffness and the system gains stability.

Though theoretically one of the stable configurations with a small mass would provide a solution to the problem, the system is still close to instability. The plot in
Figure 7.14: Plot of spring stiffness (N/m) over damper mass (kg) for horizontal damper, spinrate 75°/sec

Figure 7.15: Plot of spring stiffness (N/m) over damper mass (kg) for horizontal damper, spinrate 65°/sec
figure (7.15) shows the previously stable configuration turned into an unstable system for a lower spin rate of 65°/sec. Obviously the assumption that gravity has only a small influence on the mass only holds for small masses and high spin rates. Considering this problem and the fact that the parameters used for this analysis, especially the moments of inertia, are only estimates, it is questionable if this configuration is a desired solution. If the values for the moment of inertia turn out to different or if future modifications to the simulator change the mass properties significantly, the damper could no longer be used and a new one would have to be designed. Due to the uncertainty if stability is in fact given and the problem of unforeseen future changes to the simulator, which can drive the system to instability, this design was also dropped.

It is noteworthy at this point that the classical way to design a nutation or precession damper obviously fails, if such a system has to be designed for ground applications. In the previous approach it was tried to have the damper tuned to the resonance frequency of the system. This way the motion of the mass in the damper tube has the highest amplitude and velocity and the highest amount of energy is damped out. In order to achieve this goal the damping coefficient was held reasonably small so that the motion of the mass is not suppressed. This is illustrated in figure (7.16) in which the amount of dissipated energy is plotted over the damping ratio without the presence of gravity. To create this plot the simulation was run several times and only the damping coefficient was varied. After each simulation the difference in the total energy $\Delta E$ was calculated. It becomes obvious that the maximum of this function occurs at a low damping ratio and the curve falls rapidly if the damping is lower or higher.

Since the damper, if used on earth, can not be tuned to resonance it must be investigated if a reasonable amount of energy dissipation can be achieved for a stable configuration. The same technique as before was used and this time gravity was included in the simulation. The spring stiffness was chosen high enough to provide a stability margin. The results of this analysis is plotted in figure (7.17). The graph shows a different relationship between dissipated energy and damping ratio if gravity is present. Similar to the previous case $\Delta E$ goes to zero for low damping. If the damping ratio is very high the dissipated energy decreases again. The difference between both cases is that the dissipated energy has its maximum at much higher damping ratios which are greater than 1. The damper is overcritically damped in this case. Furthermore the graph has a very flat curvature near to its maximum which means that variations in the damping ratio do not affect the dissipated energy as significantly compared to the relation in figure (7.16) without gravity.

Contrary to the damper for space applications, which has its maximum effectivity for comparably low damping, the performance of this damper can be improved by increasing the amount of damping ratio to overcritical values. The explanation of this different functionality is, that gravity force acts on the damper mass as the simulator is tilted. Due to the precession movement the gravity force has an alternating sign along the symmetry axis of the damper and the mass is pulled back and forth through the damping fluid. In the case of the damper, which is tuned to resonance, the motion
Figure 7.16: Dissipated energy over damping ratio, without gravity

Figure 7.17: Dissipated energy over Damping ratio, with gravity
is caused by the coupling of the mass to the cage of the damper by the spring. The effect of this comparably weak coupling is suppressed if the damping is too high. In the presence of gravity the influence on the motion of the mass is higher and therefore higher damping can be chosen.

At last the influence of the other design parameters is discussed here. Since the damper is mainly driven by the gravity force acting on the mass, the effectivity increases for a heavier mass until the stability of the system is no longer provided. The distance from the center of mass does not have an influence on the gravity force. Therefore, in contrary to the classical nutation or precession damper design, the distance to the center of rotation is negligible. The plots in figure (7.18) and (7.19) show the dependence of the dissipated energy on the mass and the distance from the center of rotation. The plots were created with the same numerical analysis as before for the damping coefficient.

Simulations with estimated values for a possible damper configuration show that the affect of the damper on the simulator is significant and happens in a short amount of time. The latter fact is fortunate, since the simulations in the lab environment can only take place over a comparably short amount of time.
Figure 7.18: Dissipated energy over damper mass

Figure 7.19: Dissipated energy over distance from center of rotation
8 Design and Characterization of the Damping Mechanism

After the most promising configuration for the damper was found in the previous chapters an actual design is developed in this chapter. The input design parameters discussed in the previous chapter are again summarized and their influence on the system is briefly discussed:

**Spring stiffness**: The spring stiffness must be chosen high enough to assure stability of the system.

**Distance from center of rotation**: It became obvious from the linearized equations that the distance to the center of rotation is only influenced by the forcing function due to inertia forces. The gravitational forces are independent of this distance.

**Dampermass**: Since the motion of the mass is driven by the gravity force, the damper shows the best performance if the mass is heavy. However, stability must be given.

**Damping coefficient**: The damper dissipates the most energy for high damping ratios.

Due to the limited amount of time the result must be a concept which is easy to realize and still assures functionality by meeting the requirements for best performance listed above. Since the damper contains a fluid and is placed on the simulator near to electronic devices leakage must be avoided under all circumstances. The following requirements are made on the design:

1. The damper must be easy to manufacture.
2. Leaking must be avoided.
3. The damper must be integrable on the simulator.
4. The damping fluid should be non-toxic.
5. The damper should consist of standard parts.
6. Inexpensive solutions are preferable.

The design process took several iterative steps to find a compromise between functionality and availability as well as affordability of materials. Prior to the start of the manufacturing a design review was held in the Space Systems Simulation Lab in presence of the mechanics of the department’s workshop. The choice of materials for certain parts of the damper and improvements to the design were discussed. The necessary raw materials and parts were ordered from McMaster-Carr® and OnlineMetals[TM] and the compressable springs were ordered from CenturySpringCorporation. In the next chapters the final damper design and the implementation of Whorl-1 are described.
8.1 Design and Implementation of the Damper

A sectional view of the final damper design is shown in figure (8.20). The complete drawings used for manufacturing the damper are attached in the appendix. These drawings also include a partlist with detailed information about material and sources of all parts. The overall length of the damper is 9.5 inch (23.75 cm) and the outer diameter is 3.5 inch (8.75 cm).

For the dampertube (part 1) a translucent acrylic tube was used. This way the motion of the mass is observable when the damper is in use. The ends of the tube are closed with covers (parts 2 and 3). They are made out of Delrin (Polyacetal) which can be machined easily. One of the covers has a plug (part 11) which allows to fill in and let out the damping fluid. The covers are sealed with embedded rubber O-rings (part 5). Guidance of the mass inside the dampertube is provided by a stainless steel rod (part 6) which is centered in the two covers and threaded at the ends. The whole assembly is held together by two nuts. The rod is also sealed with small rubber O-rings. Two identical compressable springs (part 9) are used to move the mass back to the middle of the dampertube. The springs have a nominal spring stiffness of $12 \frac{N}{m}$ and have the same equal free length. In order to spare a fixing between the springs and the mass...
as well as the cover, the springs are compressed even if the mass is at the maximum excursion. This assured that they do not slide out of the centering. The mass (part 4) is made out of brass. It was originally supposed to slide on the rod with metal-to-metal contact. It turned out that the static friction was too high and the mass did not start to move during tests on the simulator. Two solutions to solve this problem were developed. The first solution, which is shown in figure (8.20), minimizes the friction between the rod and mass by using a linear ball bearing (part 12). It is centered inside the mass and is held in position by a C-ring (part 13). The bearing was manufactured by INA USA Corporation and a sectional view shown in figure (8.21). The caging of the bearing allows the balls to roll only in axial direction until they reach the end of the bearing. At that point the ball vanish inside the caging and return to the other side. The static coefficient of friction is specified to be $\mu_s = 0.004$. According to this value the mass would start to slide at an angle of $0.2^\circ$. The bearing must be lubricated in order to yield the best performance. Since the tube is totally filled with oil a good lubrication is provided.

![Figure 8.21: Linear ball bearing](image)

In the second solution a teflon tube was put around the rod and a bigger tube was centered inside the mass. The coefficient of friction between teflon and teflon is $\mu_s = 0.04$ which corresponds to an angle of $2^\circ$ degrees until the mass starts to slide. Since the first solution involving the ball bearing worked very well this solution was not realized. The complete drawings are provided in the appendix.

The damping coefficient of the damper is determined by the shape of the mass and the viscosity of the damping fluid. The easiest way to change the damping coefficient is to change the damping fluid. Therefore two different oil were purchased to determine the influence of the damping coefficient on the performance of the damper. Both oils were non-toxic and had different viscosities. More detailed specifications are given in table (8.4).

For the implementation of the damper on the simulator a simple bracket was designed. The bracket was mounted with screws onto the simulator baseplate and the damper was fastened with tube clamps. The position of the damper on the simulator is on the negative x-axis 0.42 m away from the center of rotation. A picture of the simulator with the integrated damper is shown in figure (8.22).
<table>
<thead>
<tr>
<th>type of oil</th>
<th>Viscosity (ISO-grade)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain Oil</td>
<td>32</td>
<td>McMaster-Carr (P/N 2923K27)</td>
</tr>
<tr>
<td>Gear Oil</td>
<td>100</td>
<td>McMaster-Carr (P/N 1401K21)</td>
</tr>
</tbody>
</table>

Table 8.4: Overview of used oils

Figure 8.22: Whorl-1 with the damping mechanism
8.2 Experimental Determination of Damping Coefficient

In order to characterize the damper, the different damping coefficients for the two oils must be determined. The viscosity of the oil was only specified by the manufacturer for a higher temperature than room temperature, since it is normally used in a gear or an engine which has a higher working temperature. An analytical calculation of the damping coefficient has a high uncertainty, because the viscosity at room temperature can only be roughly estimated. The easiest way to achieve a good estimate of the damping coefficient is to determine this value experimentally.

8.2.1 Physical Background

In Ref. [14], page 41 the theory of this experiment is outlined. Let us again consider the equation of motion of a damped spring mass system:

\[ \ddot{x} + 2D\omega_0 \dot{x} + \omega_0^2 x = 0. \]  

(8.186)

The damping ratio is again denoted by \( D \) and \( \omega_0 \) is the Eigenfrequency of the undamped system. The general solution to this equation is:

\[ x(t) = e^{\sigma t} \cdot (A \cos(\omega t) + B \sin(\omega t)) \]  

(8.187)

with the frequency of the damped system \( \omega = \omega_0 \sqrt{1 - D^2} \) and \( \sigma = -\omega_0 D \). \( A \) and \( B \) are constants and can be found from the initial conditions. The motion for initial conditions \( x_0 = 0 \) and \( \dot{x}_0 = v_0 \) are shown in figure (8.23).

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_0 \sqrt{1 - D^2}}, \]  

(8.188)

Figure 8.23: Motion of a damped spring mass system, D < 1

Of special interest for the experiment are the maxima in the motion of the mass, denoted in the plot with \( x_0 = x(t_0), x_1 = x(t_1), \ldots \), where time \( t_0 \) corresponds to the first maxima, \( t_1 \) to the second maximum, etc.. The period \( T \) between two maxima can be found as:
and we can write

\[ x_0 = x(t_0) \quad (8.189) \]
\[ x_1 = x(t_1) = x(t_0 + T) \]
\[ x_2 = x(t_1 + T) \]

Since \( T \) is the period of the damped system the sine- and cosine-expressions in the parantheses in equation (8.187) are equal after each period. Therefore the ratio of the values \( x_0 \) and \( x_1 \) for example only depend on the exponential term. Using the previous expressions we find:

\[ \frac{x_0}{x_1} = \frac{e^{-D\omega_0 t_0}}{e^{-D\omega_0 (t_0 + T)}} = e^{D\omega_0 T} = e^{\frac{2\pi D}{\sqrt{1 - D^2}}} . \quad (8.190) \]

For the last step the definition for the period in equation (8.188) was used. The logarithmic decrement is finally defined as:

\[ \delta = \ln \frac{x_0}{x_1} = \frac{2\pi D}{\sqrt{1 - D^2}} . \quad (8.191) \]

If the amplitude of two consecutive maxima is measured and their ratio is taken, equation (8.191) can be solved for the damping ratio using a simple rootsolver. The damping coefficient, which depends only on the shape of the mass and the viscosity of the fluid, can finally calculated from

\[ d = 2Dm\omega_0 . \quad (8.192) \]

Note that the previous equations are only valid if only viscous damping is present. In this case the logarithmic decrement is equal for all consecutive maxima. Other influences like energy dissipation caused by stretching and compressing of the spring and errors in the measurement of the position of the mass will lead to different values for \( \delta \) which allows an estimate of the accuracy of the experiment.

### 8.2.2 Setup and Execution of the Test

The equipment and setup of the test is shown in figure (8.24). The damper mass was connected to a spring with known stiffness and suspended to allow vertical oscillation inside the damper tube. The tube is completely filled with oil. The position of the mass can be measured with a graduated measuring rod. At the beginning of the experiment the stationary position of the mass was noted. The mass was excited from its stationary position as far as possible and released. After all motion was damped out and the mass returned to rest the previous step was repeated several times. In order to allow a convenient evaluation of the experiment, the motion of the mass was recorded with a digital camera. The video was later reviewed on a PC in slow motion which allowed a more precise determination of the amplitudes. The measurements are summarized in table (A.10) and table (A.11), which are attached in the appendix. All other needed values are shown in table (8.5).
8 DESIGN AND CHARACTERIZATION OF THE DAMPING MECHANISM

Table 8.5: Needed values for damping experiments

<table>
<thead>
<tr>
<th></th>
<th>low viscosity</th>
<th>high viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (kg)</td>
<td>0.805</td>
<td>0.830</td>
</tr>
<tr>
<td>spring stiffness (N/m)</td>
<td>35.4</td>
<td>35.4</td>
</tr>
</tbody>
</table>

Figure 8.24: Setup of damping experiment
8.2.3 Test Results and Discussion

**Low Viscosity** The experiment for the damping coefficient with the low viscosity oil yields three different values for the logarithmic decrement, since four maxima could be used to measure the amplitude. The values for $\delta$, $D$ and the damping coefficient are summarized in table (8.6). It becomes obvious that the value for the damping coefficient obtained from the measurements of the first two maxima differs by about 10-15% from the other values, which show a comparably high agreement. Note that the first maximum is the point at which the mass was released from rest. Several errors falsify the data obtained from that measurement. It is assumed that the initial conditions at the first maximum are: $x(t_0) = x_0$ and $\dot{x}(t = 0) = 0$. It is possible that the mass was released with some initial velocity and not from total rest. Furthermore due to the setup of the experiment the mass was lifted up by pulling it at the spring. Therefore, when it was released from rest, the part of the spring below the point it was held was relaxed and the part below was elongated by the weight of the mass. After releasing the spring itself oscillates and perturbs the motion of the mass. According to that the last two values of the damping coefficient are more likely to represent the damping.

<table>
<thead>
<tr>
<th>Logarithmic decrement $\delta$</th>
<th>1.0</th>
<th>0.88</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio $D$</td>
<td>0.16</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Damping coefficient $d$</td>
<td>1.68</td>
<td>1.47</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Table 8.6: Summary of experimental results, low viscosity

**High Viscosity** In the experiment for the damping coefficient with the high viscosity oil only one value for the logarithmic decrement could be obtained each time the mass was deviated. Since the damping is higher for this viscosity, only two maxima could be clearly measured. The values for $\delta$, $D$ and the damping coefficient are summarized in table (8.7).

The damping coefficient is as expected much higher compared to the first measurement. The disadvantage is that only a small number of data was obtainable. We already know from the previous discussion that the first measurement showed a relatively high error and the actual damping coefficient might be off by 10-15%. Nevertheless the determined value can be used as an estimate. The possible error is kept in mind if the simulation is later compared to actual measurements with the damper on the simulator.

<table>
<thead>
<tr>
<th>Logarithmic decrement $\delta$</th>
<th>3.64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping ratio $D$</td>
<td>0.50</td>
</tr>
<tr>
<td>Damping coefficient $d$</td>
<td>5.44</td>
</tr>
</tbody>
</table>

Table 8.7: Summary of experimental results, high viscosity
8.3 Characterization of Springs

The compressible springs which are used in the damper have a nominal stiffness of $c = 12 \frac{N}{m}$. The following test was done to assure that the springs indeed have the stated stiffness and to examine if the assumption of a constant value for $c$ is realistic.

8.4 Physical Background

The spring force of a linear spring is determined by

$$F_s = cx, \quad (8.193)$$

where $c$ is the spring stiffness and $x$ the deviation of the spring from its free length. The spring stiffness can experimentally found as the deviation $\Delta x$ due to the applied force $\Delta F$:

$$c = \frac{\Delta F}{\Delta x}, \quad (8.194)$$

8.4.1 Setup and Execution of the Test

The equipment and setup of the test is shown in figure (8.25). The spring is mounted vertically and masses are added on top to achieve compression. The length of the spring is then measured with a caliper. At the beginning the spring was loaded with a certain mass and its length was measured. This measurement served as the reference measurement. The spring was loaded step by step with more masses and the length was determined each time. Then the deviation $\Delta x$ and the additional load $\Delta F$ with respect to the reference measurement was calculated. The results of the measurement for two different springs are shown in the tables (A.8) and (A.9).

8.4.2 Test results and discussion

The test as described above can only be treated as a rough check of the stated spring stiffness. In order to know how accurate the method is, possible erros and their effect must be considered. The error in the measurement of the mass was determined by the scale and in the order of $\delta m = 0.1 g = 1/10000kg$. This compares to an error in the force of $\delta F = 1/1000N$. Therefore the error in the spring stiffness according to equation (8.194) is also in the same order and negligible. A much higher error is caused by the measurement error of the spring length. Since the ends of the springs are not ground, the length can only be measured with an uncertainty of $\delta l = 1mm$. For a deflection of 20 mm, the error in the spring stiffness is still about 5%. Therefore only the measurements with a deflection higher than 20 mm were considered for the determination of the stiffness.

Taking the mean of the significant values for the stiffness we yield that spring 1 has a stiffness of $c_1 = 12.1 \frac{N}{m}$ and spring 2 has a stiffness of $c_2 = 13 \frac{N}{m}$. The spring stiffness is additionally shown in figure (8.26).
8 DESIGN AND CHARACTERIZATION OF THE DAMPING MECHANISM

Figure 8.25: Setup of spring test

![Setup of spring test](image)

Figure 8.26: Plot of spring stiffness over deflection

![Plot of spring stiffness over deflection](image)
9 Measurements and Proof of Functionality

9.1 Proof of Functionality

In order to prove the functionality of the damping mechanism, the simulator with the mounted damper was trimmed as close to principal axis as possible. The center of mass was moved near to the center of rotation in order to minimize the influence of the gravitational torque. At the beginning of the experiment the simulator was set to a certain initial spin rate around the bodyfixed z-axis. Then an impulse was applied to create a precession like movement. The angular rates were measured over a interval of two minutes until the x- and y-rates were damped out. The measurements were taken with different values for spin rates for positive and negative spin. The same procedure was later repeated, but instead of the damper a ”dummy mass” was installed which has the same mass as the damper. Measurements with the same spin rates were taken to make a direct comparison between the damped and the undamped system possible. The plot in figure (9.27) shows one example of the obtained data.

In this case the oil with the ISO-grade 100 was used. The upper plot shows the graph for the spin rate around the z-axis. Both experiments started at the same initial spin rate and they decrease equally which is due to the resistance of the spinning simulator in the air. It becomes obvious that the z-rates are not significantly affected by the damping mechanism. The lower plot shows the graphs for the x- and y-rates.

![Comparison of the angular rates for the simulator with and without the damper](image1)

![Comparison of the angular rates for the simulator with and without the damper](image2)

Figure 9.27: Comparison between the damped and undamped system
The plots show that the amplitudes of the x- and y-rates of the damped and the undamped system are almost equal. In fact, the undamped system starts with slightly lower rates. Nevertheless, the undesired motion of the simulator is damped out in about 80 seconds. After that, the rates remain in the order of $0.5^\circ$/sec, which is due to the fact that the simulator is not trimmed exactly to principal axis and has therefore a certain amount of self-induced ‘wobble’. The rates for the undamped system also decrease over time due to the influence of drag, but it takes significantly longer. The same results were consistent through all experiments with different rates and directions and the influence of the damper was always visible.

### 9.2 Comparison between Damping Fluids

After the functionality of the damper was proven, the influence of a different damping coefficient is examined in a second experiment. The measurements were taken following the same procedure as before only the oil in the damper was replaced. Since the damping coefficient is lower in this case a lower effect of the damper is expected. The plot in figure (9.28) shows a comparison between the oil with the low and the high viscosity at an initial spin rate of $-71^\circ$/sec. Though it is visible that the damping is smaller

![Comparison of the angular rates for the simulator different damping coefficients](image)

![Comparison of the angular rates in °/sec](image)

Figure 9.28: Comparison between different oils

for the oil with the lower viscosity (ISO-grade 32), a higher difference in the damping effect was expected. An explanation for this unexpected result is that the friction in
the bearing affects the motion of the mass. The friction force also causes damping similar to the viscous force, but it is not proportional to the velocity. According to the experiments in which the damping coefficients were estimated, the damping ratios due to viscous damping are \( D \simeq 0.5 \) and \( D \simeq 0.14 \), which would correspond to the far left side of the graph in figure (7.17). Obviously the additional energy dissipation caused by friction causes a higher total damping ratio near to \( D = 1 \). Since the slope of the graph becomes more flat in this region the same difference in the damping ratio leads to a smaller difference in the dissipated energy, which explains the small difference in the damping effect thought the oils had different viscosities.

The two plots discussed in this chapter are representative for all measurements. For reference all plots of the measured data for both experiment are attached in the appendix.
10 Comparison of Measurements and Simulated Data

The comparison between the actual measurements and the software simulation depends of course on the accuracy of the input parameters which describe the system. Furthermore it must be assured that all influences on the system are modelled correctly in the simulation. This requirement is not trivial, since in every simulation assumption to simplify the given problem are made and it can mostly only found out experimentally if the assumptions hold for all cases. Sometimes not all influences on the system are known up front and can therefore not be included in the simulation. The following chapters document which steps must be taken to make the simulation fit the measurements.

10.1 Drag Modell

The first step to make the simulation and the measurements comparable is the inclusion of a drag model. An estimation of the perturbations which act on the simulator is given in reference [15]. The drag due to the resistance in the air turned out to be superior to all other perturbations. The drag torque was simulated according to a very simple model. The drag force due to a small cross-sectional area element $A_i$ is given by:

$$D_i = C_D \frac{\rho_{\text{air}}}{2} u_i^2 A_i,$$

(10.195)

where $C_D$ is the drag coefficient, $\rho_{\text{air}}$ is the density of air, and $u_i$ is the velocity of this small area element. A drag coefficient of $C_D = 1$ was assumed and $\rho_{\text{air}}$ was set to $1.225 \text{kg/m}^3$. Due to the rotation of the simulator the velocity is equal to $u_i = \omega r_i$, where $r_i$ is the distance to the area element from the center of rotation. The drag torque of each area element then found by:

$$L_i = r_i \cdot D_i = C_D \frac{\rho_{\text{air}}}{2} \omega^2 r_i^3 A_i.$$

(10.196)

To find the total drag torque we must sum up over the total cross-sectional area of the simulator. We find for the drag moment around the z-axis:

$$L_z = \sum_{r_i=0}^{r_{\text{sim}}} L_i = C_D \frac{\rho_{\text{air}}}{2} \omega^2 r_{\text{sim}}^3 A_z = C_D \frac{\rho_{\text{air}}}{2} \omega^2 r_{\text{sim}}^4 h_z.$$

(10.197)

In the last step we substitute $A_z = r_{\text{sim}} h_z$, where $h_z$ is the average height of the components of the simulator. In a similar way we obtain for the drag around the x- and y-axis:

$$L_{xy} = \sum_{r_i=0}^{r_{\text{sim}}} L_i = C_D \frac{\rho_{\text{air}}}{2} \omega^2 r_{\text{sim}}^3 A_{xy} = C_D \frac{\rho_{\text{air}}}{2} \omega^2 r_{\text{sim}}^4 A_{sim}.$$

(10.198)

In this equation $A_{\text{sim}}$ is the area of the simulator base plate. The drag torque vector is implemented in the equations of motion in the equation for $B_\mathbf{H}$. Note that this model showed the expected results very for the torque around the z-axis. However the torques around the x- and y-axis given in equation (10.198) need to be scaled by a factor of 10 to fit to the measured results.
10.2 Mass Properties of the Simulator

Since the configuration of the simulator was modified while it was trimmed to principal axis, the old mass properties which have been determined with a comparably high accuracy did no longer apply. As a first estimate, the mass properties for the simulation were taken from an updated version of the CAD-modell. It turned out that the simulation differed significantly if the products of inertia from the CAD-modell were used. In the simulation the major axis differed from the body-frame axis and the simulator ended up in a simple spin with a high tilt angle, which exceeded the allowed tilt angle of the air bearing. In the experiments Whorl-1 always returned to a spin roughly about its z-axis and even without the damper the simulator showed only very little wobble while spinning about its z-axis. The latter observation indicates that the principle axes are close the body-fixed axes.

In order to achieve higher accuracies, the mass properties of the simulator must be found experimentally. A method for this experiment involving Kalman filtering is developed in reference [15], but knowledge about the attitude of the simulator is necessary in order to achieve better estimates. With the actual sensor equipment of Whorl-1 only one vector measurement, the direction of earth’s gravity field, can be obtained. Since two vectors are needed to calculate the attitude, the implementation of a second sensor, a magnetometer, is in progress.

A different method to obtain better estimates of the mass properties based on a least-squares-fit was tried. The angular accelerations of the simulator can be calculated using EULER’s rotational equations:

\[ \dot{\omega} = I^{-1}(-\omega \times L\omega - L), \]  

(10.199)

where \( L \) is the drag torque as derived in the previous chapter. The measured angular rates are the input values for this equation to calculate the angular accelerations at each time step of the measurement. The values for \( \dot{\omega} \) can also be obtained from numerically differentiating the data for the angular velocities. Since differentiation of noisy data is always unstable, the data must be filtered previously. The MATLAB filter function 'filter' was used in the script 'datafilter.m' which was written for that purpose. The script takes the two previous and the two following datapoints together with the actual data point and takes the average according to:

\[ \tilde{y}(t_n) = \frac{1}{5}y(t_{n-2}) + \frac{1}{5}y(t_{n-1}) + \frac{1}{5}y(t_n) + \frac{1}{5}y(t_{n+1}) + \frac{1}{5}y(t_{n+2}). \]

The smoothed data curve for \( \omega \) can now differentiated and more stable results are gained. The square of the difference between the measured data \( \tilde{\omega} \) and the calculated data \( \dot{\omega} \) from equation (10.199) can be calculated in a cost function \( R \):

\[ R = (\tilde{\omega} - \dot{\omega})^2. \]  

(10.200)

In the least squares method it is tried to minimize the value of the cost function by varying the components of the inertia matrix. It was tried to develop an optimization
algorithm which lead to lengthy equations which could not be used. The built in least-squares-fit in the program 'Mathematica' ran into singularity issues. Since better estimates could not be obtained at the moment the estimates of the CAD-modell for the moments of inertia were used. The products of inertia were reduced by a factor of ten. The results of the simulation do not fit exactly to the measured results this way but the simulation comes reasonably close and is at least qualitatively comparable.

10.3 Friction Modell

As already mentioned in chapter 9 the plots for the angular rates obtained with the simulation for the oil with the low viscosity did not match with the measured results. In the simulation without the friction modell the oil with the ISO-grade 32 does not damp out the x- and y-rates as fast as found in the experiment. A better agreement can be found if a simple friction modell is implemented. The static and dynamic coefficient of friction in the bearing is estimated by the manufacturer as $\mu = 0.004$. This value is reasonable since the mass already starts to move for angles $< 1^\circ$. Since the friction force is proportional to the normal force caused between the rod and the mass we find the following estimate of the friction:

$$F = \mu N = \mu mg, \quad (10.201)$$

where $mg$ is the gravity force of the damper mass. Since we assume small nutation angles the influence of the tilt angle is negligible and the full weight of the mass is assumed to act on the rod. With this simple modell we can estimate the friction to be in the order of $F = 0.03$ N. Including the friction force as an additional force in the equation for $B_{p_d,n}$ yields significantly different results in the simulation. The plots in figure (10.29) shows the simulated data for the oil with ISO-grade 100. The simulation was run twice, the first time without the friction modell and then including the bearing friction. The angular rates are plotted in the same diagram for comparison. It becomes obvious that the friction does not affect the angular rates much, though it is obtainable that the rates for the simulation with the friction modell damp out faster. However both graphs are similar and the rates are mainly damped out after 80 seconds in both cases. The plots for this damping fluid show a high agreement with the data from the experiment and the necessity of including friction was not recognizable. Note that to create the graph without friction the damping coefficient of $d = 5.4Ns/m$ was used, which was found in the first experiment for the damping coefficient. It turned out after the second experiment that this value is probably 10%-15% to high (see chapter 8). Therefore for the graph with the friction modell the damping coefficient was reduced by 10%. It becomes obvious now that the value for the damping coefficient, which was determined to high, covered up the influence of friction.

In figure (10.30) the results of the simulation for the damping fluid with ISO-grade 32 is shown. Again the simulation was run twice, the first time without the friction modell and then including the bearing friction. The damping coefficient was equal in both cases since it was in this case determined with a higher accuracy. In this plot the effect of the friction becomes visible since the x- and y-rates are damped out after
80 seconds for the simulation including the friction model. This graph shows a high agreement with the experimental results. On the other hand without the bearing friction the angular velocities are still comparably high after even after 120 seconds which does not happen in reality.

Finally the plot in figure (10.31) shows a direct comparison between the angular rates for the two damping fluids with the friction model. If this numerically simulated data is compared with the measured data for the different damping fluids, it can be seen that the data is qualitatively comparable and the implementation of the friction model lead to an improvement of the simulation.

In summary the drag model and the friction model are necessary corrections to make the simulation more close to the measured results. Further improvement can only be gained if the mass properties for Whorl-1 are determined with higher accuracy. Since the moments and products of inertia have a significant influence on the dynamic behaviour of the system and all main sources if friction and external forces are accounted for, the errors in these value are the main error source by now. It can be expected that if the mass properties are available in the future, the drag model and friction model must be reevaluated.

![Comparison of simulation with and without friction model](image1)

![Plot of angular rates over time](image2)

Figure 10.29: Comparison of the simulation for high viscosity oil with and without friction
Figure 10.30: Comparison of the simulation for low viscosity oil with and without friction

Figure 10.31: Comparison of angular rates for different damping fluids with friction
11 Conclusions

At the beginning of this research project the equations of motion are derived for a model of the simulator with the attached spring mass damper. The simulator is assumed to be a rigid body, which is a valid assumption often used for small satellites or spacecraft without moving parts. The damper mass is assumed to be a point mass. At the beginning the damping was assumed to be only viscous damping. It turned out that also friction must be included into the damping. The equations of motion are derived in the most general way to make the simulation as flexible as possible.

The numerical simulation is written in MATLAB and consists of three main scripts. The core script handles the input and output of data and calls the MATLAB integrator ODE45 to integrate the equations of motion. The initial conditions of the entire simulation are summarized in a config-file, which allows to save and reproduce certain configurations of the system. The debugging of the simulation is accomplished by calculating the time history of the kinetic energy by integration and later analytically with the help of the integrated values for the angular rates and the velocity of the mass. It turned out that these values differ significantly if errors in the simulation occur or if the integration accuracy is chosen to low.

The equations of motion are linearized to derive the stability criteria for two different configurations: the horizontal damper and the vertical damper. The criteria of Hurwitz are used to find the conditions for which the system is stable. In both cases stability is only given if the system spins around the major axis. Furthermore a condition for the minimal required spring stiffness is found which depends on the damper mass, the mass properties of the simulator, the spin rate and the distance from the center of rotation. This condition is different for the vertical and the horizontal configuration and depended on different values.

In order to achieve the highest energy dissipation rate, the conditions for an optimal tuned damper are developed from the linearized equations. The classical approach for damper systems in space is to tune the damper to the frequency of its forcing function to achieve resonance. For the vertical damper (precession damper) this frequency is the precession frequency. The horizontal damper (nutation damper) must be tuned to twice the precession frequency. The results are verified with the help of the simulation. It is observed that both dampers are sensitive to detuning and the energy dissipation rate decreases significantly in this case.

It is examined of the concepts for the tuned precession and nutation damper are feasible on earth. It turns out that the precession damper is stable but the required, low spring stiffness makes a practical realization on earth impossible. The nutation damper is on earth only stable for a light damper mass and high spin rates. The realized design for a damping mechanism on earth differs from the classical design. In order to achieve stability the spring stiffness is chosen sufficiently high. Numerical analysis shows that higher energy dissipation rates can be reached on earth by a high damping ratio. The
developed damping mechanism is therefore not called nutation damper since it works according to a different concept.

The damper consists of an acrylic tube filled completely with a damping fluid. The damper mass slides inside the tube on a rod and is returned to its middle position by two compressible springs. A linear ball bearing reduces the friction between the rod and the mass. The damper is implemented horizontally on the simulator’s base plate. The springs for the damper are examined if nonlinear behaviour occurs and the damping coefficient for viscous damping is determined experimentally.

In order to take measurements with the implemented damper the simulator is trimmed close to principal axes. Measurements with and without the damper are taken and the functionality of the damper is proven. The undesired angular rates around the x- and y-axis are damped out after about 80 seconds, while dampers in space need several minutes to return the spacecraft to a simple spin (ref [16]). The fast damping of the developed mechanism is beneficial for simulations in a laboratory environment. The measurements are repeated with a different damping fluid with lower viscosity. The damping effect with the second fluid is higher than predicted with the simulation, which leads to the conclusion that friction in the bearing has a significant effect on the damping.

The comparison between the simulation and the measurements shows that a drag modell and a friction modell must be included in the simulation. The remaining error cannot be removed at the moment since the mass properties of the simulator cannot be estimated accurate enough without the ability to measure the attitude of the simulator. Therefore the implementation of a magnetometer as a vector sensor is in progress. Other error sources are for example that the effective damper mass is lower than the actual mass since it is surrounded by oil which creates a certain lift. Furthermore the spring configuration in the damper consists of two compressible springs with a slightly different spring stiffness. In the simulation these springs are substituted with a single spring with twice the spring stiffness as the actual springs. In theory this can only be done if both springs have absolute equal properties. The influence of these errors is assumed to be small compared to the error caused by the inaccurate mass properties. Despite this remaining error the simulation produces results which are comparable to the experimental data.
12 Future work

With this research project the foundation for further activities in the field of passive attitude control in the systems simulation lab was layed. It was proven that the damper is functional and damps out the undesired coning motion of the simulator in a short amount of time which is advantageous for experiments in a laboratory environment. Some ideas for future work and research involving the damper is given in this chapter.

The uncertainty in the mass properties of the simulator turned out to be a major problem if the measurements from the simulator are compared to the MATLAB-simulation. The existing algorithms to determine the mass properties work only if the attitude of the simulator can be determined. Efforts which were done to yield better estimates of the mass properties with the existing sensors remained unsuccessful. The development of an attitude sensor can solve this problem.

The simulator with the attached damper will end up in a major axis spin after the coning motion is damped out. If this axis is not aligned with the body-fixed z-axis, which points normal to the simulator base plate, the simulator will spin in a tilted orientation. In order to keep this tilt angle small the simulator must be trimmed so that the body-fixed z-axis as close to the major axis as possible. For this research project the method of choice was to remove components which were not required and reduce the products of inertia this way. In the future a more sophisticated method is preferrable which allows to keep the full configuration of Whorl-1. One solution to this problem is to provide more mountings so that trim masses can be added more easily.

The PhD-dissertation of Ralph A. Sandfry (ref [8]) examined the equilibria of a rigid body with an attached reaction wheel and a point mass damper. A system with this configuration is now available in the Space Systems Simulation Lab. In the future the results found in reference [8] can be verified experimentally.

Different maneuvers can be performed and the effect of the damper can be examined. A possibility for such a maneuver is simply to spin up the reaction wheel aligned with the z-axis. The simulator will then start to spin in the opposite direction and since the body-fixed axes are not perfectly aligned with the principal axes ‘wobble’ will occur. This maneuver can be done with and without the damping mechanism and differences can be examined.

Finally, as already mentioned at the beginning of this report, the spring-mass-damper configuration is only one out of a variety of damping mechanisms. A different damper type could be realized and compared with the existing one. A ring nutation damper appears to be a promising alternative. If the ring is totally filled with fluid the center of mass of this damper does not vary. This way gravitational torques do not occur which is an advantage compared to a damper with moving parts.
References


A Appendix

A.1 Rotation matrices

The rotation matrices are defined as a positive rotation by the angle $\alpha$. The index denotes the rotation axis.

Rotation around the 1-axis:

$$R_1(\alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_1 & \sin \alpha_1 \\ 0 & -\sin \alpha_1 & \cos \alpha_1 \end{bmatrix}$$  \hspace{1cm} (A.202)

Rotation around the 2-axis:

$$R_2(\alpha_2) = \begin{bmatrix} \cos \alpha_2 & 0 & -\sin \alpha_2 \\ 0 & 1 & 0 \\ \sin \alpha_2 & 0 & \cos \alpha_2 \end{bmatrix}$$  \hspace{1cm} (A.203)

Rotation around the 3-axis:

$$R_3(\alpha_3) = \begin{bmatrix} \cos \alpha_3 & \sin \alpha_3 & 0 \\ -\sin \alpha_3 & \cos \alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (A.204)
### A.2 Spring characterization

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Δ Force (N)</th>
<th>length (mm)</th>
<th>deflection (mm)</th>
<th>stiffness (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.30</td>
<td>-</td>
<td>100.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18.10</td>
<td>0.06</td>
<td>95.00</td>
<td>5.00</td>
<td>11.38</td>
</tr>
<tr>
<td>23.00</td>
<td>0.10</td>
<td>91.00</td>
<td>9.00</td>
<td>11.66</td>
</tr>
<tr>
<td>27.80</td>
<td>0.15</td>
<td>86.00</td>
<td>14.00</td>
<td>10.86</td>
</tr>
<tr>
<td>33.60</td>
<td>0.21</td>
<td>80.00</td>
<td>20.00</td>
<td>10.45</td>
</tr>
<tr>
<td>40.20</td>
<td>0.27</td>
<td>77.00</td>
<td>23.00</td>
<td>11.90</td>
</tr>
<tr>
<td>46.20</td>
<td>0.33</td>
<td>70.00</td>
<td>30.00</td>
<td>11.09</td>
</tr>
<tr>
<td>52.00</td>
<td>0.39</td>
<td>68.00</td>
<td>32.00</td>
<td>12.17</td>
</tr>
<tr>
<td>58.80</td>
<td>0.46</td>
<td>60.00</td>
<td>40.00</td>
<td>11.40</td>
</tr>
<tr>
<td>66.50</td>
<td>0.53</td>
<td>56.00</td>
<td>44.00</td>
<td>12.08</td>
</tr>
<tr>
<td>73.40</td>
<td>0.60</td>
<td>50.00</td>
<td>50.00</td>
<td>11.99</td>
</tr>
<tr>
<td>79.10</td>
<td>0.66</td>
<td>47.00</td>
<td>53.00</td>
<td>12.36</td>
</tr>
<tr>
<td>85.00</td>
<td>0.71</td>
<td>40.00</td>
<td>60.00</td>
<td>11.89</td>
</tr>
<tr>
<td>91.70</td>
<td>0.78</td>
<td>36.00</td>
<td>64.00</td>
<td>12.17</td>
</tr>
<tr>
<td>97.40</td>
<td>0.83</td>
<td>31.00</td>
<td>69.00</td>
<td>12.10</td>
</tr>
<tr>
<td>103.90</td>
<td>0.90</td>
<td>29.00</td>
<td>71.00</td>
<td>12.66</td>
</tr>
<tr>
<td>108.40</td>
<td>0.94</td>
<td>25.00</td>
<td>75.00</td>
<td>12.57</td>
</tr>
<tr>
<td>115.10</td>
<td>1.01</td>
<td>22.00</td>
<td>78.00</td>
<td>12.93</td>
</tr>
<tr>
<td>120.50</td>
<td>1.06</td>
<td>19.00</td>
<td>81.00</td>
<td>13.10</td>
</tr>
</tbody>
</table>

Table A.8: Testresults for spring 1
<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>∆ Force (N)</th>
<th>length (mm)</th>
<th>deflection (mm)</th>
<th>stiffness (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.10</td>
<td>-</td>
<td>105.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18.00</td>
<td>0.06</td>
<td>100.00</td>
<td>5.00</td>
<td>11.58</td>
</tr>
<tr>
<td>23.10</td>
<td>0.11</td>
<td>96.00</td>
<td>9.00</td>
<td>11.99</td>
</tr>
<tr>
<td>28.90</td>
<td>0.16</td>
<td>93.00</td>
<td>12.00</td>
<td>13.73</td>
</tr>
<tr>
<td>35.20</td>
<td>0.23</td>
<td>88.00</td>
<td>17.00</td>
<td>13.33</td>
</tr>
<tr>
<td>41.30</td>
<td>0.29</td>
<td>82.00</td>
<td>23.00</td>
<td>12.45</td>
</tr>
<tr>
<td>47.80</td>
<td>0.35</td>
<td>77.00</td>
<td>28.00</td>
<td>12.51</td>
</tr>
<tr>
<td>53.70</td>
<td>0.41</td>
<td>74.00</td>
<td>31.00</td>
<td>13.16</td>
</tr>
<tr>
<td>59.50</td>
<td>0.46</td>
<td>69.00</td>
<td>36.00</td>
<td>12.92</td>
</tr>
<tr>
<td>65.30</td>
<td>0.52</td>
<td>64.00</td>
<td>41.00</td>
<td>12.73</td>
</tr>
<tr>
<td>74.10</td>
<td>0.61</td>
<td>58.00</td>
<td>47.00</td>
<td>12.94</td>
</tr>
<tr>
<td>80.00</td>
<td>0.67</td>
<td>54.00</td>
<td>51.00</td>
<td>13.06</td>
</tr>
<tr>
<td>85.20</td>
<td>0.72</td>
<td>49.00</td>
<td>56.00</td>
<td>12.81</td>
</tr>
<tr>
<td>91.10</td>
<td>0.77</td>
<td>45.00</td>
<td>60.00</td>
<td>12.92</td>
</tr>
<tr>
<td>97.80</td>
<td>0.84</td>
<td>41.00</td>
<td>64.00</td>
<td>13.14</td>
</tr>
<tr>
<td>103.70</td>
<td>0.90</td>
<td>37.00</td>
<td>68.00</td>
<td>13.21</td>
</tr>
<tr>
<td>110.40</td>
<td>0.96</td>
<td>30.00</td>
<td>75.00</td>
<td>12.86</td>
</tr>
<tr>
<td>116.40</td>
<td>1.02</td>
<td>28.00</td>
<td>77.00</td>
<td>13.29</td>
</tr>
<tr>
<td>122.60</td>
<td>1.08</td>
<td>25.00</td>
<td>80.00</td>
<td>13.55</td>
</tr>
</tbody>
</table>

Table A.9: Test results for spring 2

### A.3 Experimental determination of damping coefficient

<table>
<thead>
<tr>
<th>No.</th>
<th>(x_0) (inch)</th>
<th>(x_1) (inch)</th>
<th>(x_2) (inch)</th>
<th>(x_3) (inch)</th>
<th>(x_\infty) (inch)</th>
<th>(\delta_0) ((\cdot))</th>
<th>(\delta_2) ((\cdot))</th>
<th>(\delta_3) ((\cdot))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.0000</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0169</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>2</td>
<td>13.0000</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0169</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>3</td>
<td>12.8750</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>0.9734</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>4</td>
<td>12.8750</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>0.9734</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>5</td>
<td>12.8750</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>0.9734</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>6</td>
<td>12.7500</td>
<td>11.0625</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>0.9886</td>
<td>0.8267</td>
<td>0.8473</td>
</tr>
<tr>
<td>7</td>
<td>13.0000</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0169</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>8</td>
<td>12.8750</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>0.9734</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>9</td>
<td>12.8750</td>
<td>11.0625</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0341</td>
<td>0.8267</td>
<td>0.8473</td>
</tr>
<tr>
<td>10</td>
<td>13.0000</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0169</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
<tr>
<td>11</td>
<td>13.0000</td>
<td>11.1250</td>
<td>10.5000</td>
<td>10.2500</td>
<td>10.0625</td>
<td>1.0169</td>
<td>0.8873</td>
<td>0.8473</td>
</tr>
</tbody>
</table>

\[\delta\]
\[\bar{D}\]

Table A.10: Test results for oil with low viscosity
Table A.11: Test results for oil with high viscosity

<table>
<thead>
<tr>
<th>No.</th>
<th>$x_0$ (inch)</th>
<th>$x_1$ (inch)</th>
<th>$x_\infty$ (inch)</th>
<th>$\delta_0$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.8750</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.5835</td>
</tr>
<tr>
<td>2</td>
<td>11.8750</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.5835</td>
</tr>
<tr>
<td>3</td>
<td>12.0000</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.6276</td>
</tr>
<tr>
<td>4</td>
<td>12.0000</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.6376</td>
</tr>
<tr>
<td>5</td>
<td>12.0000</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.6376</td>
</tr>
<tr>
<td>6</td>
<td>12.0000</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.6376</td>
</tr>
<tr>
<td>7</td>
<td>12.2500</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.7377</td>
</tr>
<tr>
<td>8</td>
<td>12.2500</td>
<td>9.6875</td>
<td>9.6250</td>
<td>3.7377</td>
</tr>
</tbody>
</table>

\[ \delta = 3.64 \]
\[ \bar{D} = 0.50 \]

A.4 Code of Numerical Simulation

A.4.1 Code of ’config.m’

clear
% define degree conversion
deg2rad = pi/180;

% time interval of integration
tspan = [0 120];

% gravity
g = [0; 0; -9.81];

% positionvector of point O' in inertial frame
R_O = [0; 0; 0.8];

% mass of simulator
M = 84;

% dampermass
m = 0.8;

% spring constant
c = 12.5*2;

% damper constant
d = 5;

% coefficient of friction
mu = 0.004;
% inertia matrix of whole system about CR
R_whorl_cad = [0 -1 0; 1 0 0; 0 0 1]* [1 0 0; 0 -1 0; 0 0 -1];
I_cad = [ 7.476 -0.06057 -0.01654;
         -0.06057 6.4233 0.01482;
         -0.01654 0.01482 12.2906];
I = R_whorl_cad*I_cad*R_whorl_cad’

% initial position and velocity of dampermass
u0 = 0;
v0 = 0;

% orientation of the damper
n = [0; 1; 0];
n = n/norm(n);

% position of dampermass for relaxed spring in body frame
r_d0 = [-0.42; 0; 0.1];

% initial damperposition
r_d = r_d0 + u0*n;

% center of gravity of simulator without dampermass in body-frame relative to 0
r_cm = -m/M*r_d0 + [0; 0; -0.01];

% initial conditions for the simulator
q0 = [0; 0; 0; 1];
omega0 = [-5.5; -2; -76]*deg2rad;

% Thruster initial conditions
F_th = [0; 0; 0];
r_th = [0; 0; 0];

% intertia matrix of only the simulator
% since I_cm has the dampermass in it, subtract it here
Isim = I + m*squewm(t(r_d))*squewm(t(r_d));

% initial conditions for linear momentum and angular momentum
H0 = I*omega0 + m*v0*cross(r_d,n);
p_d0 = m*(v0 - n'*squewm(t(r_d))*omega0);

% initial kinetic energy
T_0 = 0.5*omega0’*I*omega0 + 0.5*m*v0^2 + m*v0*omega0’*cross(r_d,n);

% state vector
y0 = [q0; H0; p_d0; u0; T_0];

% density of air
rho_air = 1.225;

% radius of simulator for drag modell
r_sim = 0.50;

% area in z-direction for drag modell
Az = 0.2;

% area in x-y-direction for drag modell
Axy = pi*r_sim^2;

% drag coefficient
C_d = 1;

% combine constant for drag modell
drag = [C_d,r_sim,rho_air,Az,Axy];

% calculated precession frequency
p = sqrt((H0'*H0-2*I(1,1)*T_0)*(I(3,3)-I(2,2))/(I(1,1)*I(3,3)*I(2,2)));
fprintf(['\n Initial precession frequency p = ',num2str(p),' 1/sec.\n']);

A.4.2 Code of 'iohandler.m'

% read in config file
config

% get constant crossprod out of interation (3x1 vector)
crossprod = cross(r_d,n);

fprintf('\n Starting integration! \n');

% initialize integration
options=odeset('reltol',1e-5,'abstol',1e-5);
[t,y] = ode45(@equofmo,tspan,y0,options,Isim,M,m,r_cm,r_d0,c,d,n,r_th,
F_th,crossprod,g,drag,mu);
% extract the solutions from varaible y
q = y(:,1:3)';
q4 = y(:,4)';
H = y(:,5:7)';
p = y(:,8)';
u = y(:,9)';
T_numeric = y(:,10)';
fprintf(‘\n Integration is done! \n’);

fprintf(‘\n Enter loop! \n’);

% calculate omega, v, kinetic and potential energy for every timestep
for i = 1:size(p,2)
    r_d(:,i) = r_d0 + u(i)*n;
    I(:,i) = Isim - m*squewmat(r_d(:,i))*squewmat(r_d(:,i));

    aa = [H(:,i); p(i)];
    CC = [I(:,i) m*crossprod;
          m*crossprod' m         ];
    bb = CC\aa;

    omega(:,i) = bb(1:3);
    v(i) = bb(4);

    q_test(i) = norm([q(:,i); q4(i)]);
    R_bn(:,i) = (q4(i)^2 - q(:,i)'*q(:,i))*eye(3) + 2*(q(:,i)*q(:,i)'-q4(i)*squewmat(q(:,i)));
    nutationangle(i) = acos(R_bn(3,3,i));

    T_analyt(i) = 0.5*omega(:,i)'*(I(:,i)*omega(:,i)) + 0.5*m*v(i)^2 + m*v(i)*omega(:,i)'*crossprod;
    V(i) = - M*g'*(R_O + R_bn(:,i)'*r_cm) - m*g'*(R_O + R_bn(:,i)'*r_d(:,i)) + 0.5*c*u(i)^2;
end

fprintf(‘\n Out of loop! \n’);

% calculate control values
deltaT = T_analyt - T_numeric;
E_numeric = (V + T_numeric);
E_analyt = (V + T_analyt);

delta_E = E_numeric(1) - E_numeric(size(p,2))
fprintf([‘\n Done with simulation for d = ,num2str(d),’ N/m^2\n’]);
figure(1);
plot(omega(1,:),omega(2,:))
xlabel(‘\omega_2’);
ylabel(‘\omega_1’);
title([‘Plot of \omega_1 over \omega_2 for m = ,num2str(m),’ kg,
        d = ,num2str(d),’ Nsec/m, c = ,num2str(c),’ N/m^2’]);
grid on;
A.4.3 Code of 'equofmo.m'

function dydt = equofmo(t,y,Isim,M,m,r_cm,r_d0,c,d,n,r_th,F_th,crossprod,g,drag,mu)

% extract the initial conditions
q = y(1:3);
q4 = y(4);
H = y(5:7);
p_d = y(8);
u = y(9);
T = y(10);

% calculate the inertia matrix
r_d = r_d0 + u*n;
I = Isim - m*squewmat(r_d)*squewmat(r_d);

% find the rotation matrix from n to b frame
R_bn = (q4^2 - q'*q)*eye(3) + 2 *(q*q'-q4*squewmat(q));

% extract the velocity and omega form H and op
aa = [H; p_d];
CC = [I m*crossprod;
      m*crossprod' m ];
bb = CC\aa;

omega = bb(1:3);
v = bb(4);

% compute friction force
friction = mu*m*9.81*sign(v);

% compute the drag torque
C_d = drag(1);
r_sim = drag(2);
rho_air = drag(3);
Az = drag(4);
Axy = drag(5);
dragtorque = 0.5*C_d*rho_air*r_sim^3*[10*Axy*omega(1)^2*sign(omega(1));
       10*Axy*omega(2)^2*sign(omega(2)); Az*omega(3)^2*sign(omega(3))];

% set up differential equations
q_dot = 0.5 * [(squewmat(q) + q4 * eye(3,3)); -q'] * omega;

H_dot = -cross(omega,H) + M*cross(r_cm,R_bn*g) + m*cross(r_d,R_bn*g)
\[ + \text{cross}(r\_th,F\_th) - \text{dragtorque}; \]

\[ p\_d\_dot = -n'\text{cross}(\text{omega},m*(v*n+\text{cross}(\text{omega},r\_d))) - c*u - d*v \]
\[ + m*n'(R\_bn*g) - \text{friction}; \]

\[ u\_dot = v; \]

\[ T\_dot = \text{omega}'*(M*\text{cross}(r\_cm,R\_bn*g) + m*\text{cross}(r\_d,R\_bn*g) + \text{cross}(r\_th,F\_th)) \]
\[ + v*m*(R\_bn*g)'*n + v*(-c*u-d*v) - \text{omega}'*\text{dragtorque} - v*\text{friction}; \]

\[ \text{dydt} = [q\_dot; H\_dot; p\_d\_dot; u\_dot; T\_dot]; \]

A.4.4 Code of ’makeplots.m’

figure(1);
subplot(2,1,1);
plot(t,u);
title('Plot of relative position u of damper mass over time');
grid on;
subplot(2,1,2);
plot(t,[q; q4]);
title('Plot of quaternions over time');
legend('q1','q2','q3','q4',0);
grid on;

figure(2);
subplot(2,1,1);
plot(t,omega(3,:)/deg2rad);
title('Plot of \omega_z over time');
legend('\omega_z',0);
grid on;
subplot(2,1,2);
plot(t,omega(1:2,:)/deg2rad);
title('Plot of angular rates over time');
legend('\omega_x','\omega_y',0);
grid on;

figure(4);
plot(t,E\_numeric,t,E\_analyt);
title('Plot of E numeric and E analyt');
legend('E numeric','E analyt');
grid on;

figure(5);
plot(t,q\_test);
title('Plot of |q| over time');
grid on;

figure(6);
plot(t,deltaT)
title('Plot of (T analyt - T numeric)');
grid on;
A.5 Technical Drawings

Figure A.32: Technical drawings for bearing version, sheet 1
Figure A.33: Technical drawings for bearing version, sheet 2
Figure A.34: Technical drawings for bearing version, sheet 3
Figure A.35: Technical drawings for bearing version, sheet 4
Figure A.36: Technical drawings for bearing version, sheet 5
Figure A.37: Technical drawings for bearing version, sheet 6
<table>
<thead>
<tr>
<th>Part-No</th>
<th>Title</th>
<th>Material</th>
<th>Amount</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Damper tube</td>
<td>Cast Acrylic</td>
<td>1</td>
<td>McMaster-Carr F/N 8668K557</td>
</tr>
<tr>
<td>2</td>
<td>Cover</td>
<td>Delrin</td>
<td>1</td>
<td>McMaster-Corr F/N 8576K35</td>
</tr>
<tr>
<td>3</td>
<td>Cover</td>
<td>Delrin</td>
<td>1</td>
<td>McMaster-Corr F/N 8576K35</td>
</tr>
<tr>
<td>4</td>
<td>Mass</td>
<td>Brass 360 H92</td>
<td>1</td>
<td>Online Metals</td>
</tr>
<tr>
<td>5</td>
<td>O-Ring</td>
<td>Rubber</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Rod</td>
<td>Stainless Steel T-304</td>
<td>1</td>
<td>Online Metals</td>
</tr>
<tr>
<td>7</td>
<td>Washer</td>
<td>Iron</td>
<td>2</td>
<td>Century Spring Corp. F/N 11669</td>
</tr>
<tr>
<td>8</td>
<td>Spring</td>
<td>Iron</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Linear ball bearing</td>
<td>--</td>
<td>1</td>
<td>INA F/N KE208PPS175348</td>
</tr>
<tr>
<td>10</td>
<td>O-Ring</td>
<td>Rubber</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Plug</td>
<td>Brass</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>C-Ring</td>
<td>Steel plate</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Nut</td>
<td>Iron</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
Figure A.39: Technical drawings for tube version, sheet 1
Figure A.40: Technical drawings for tube version, sheet 2
Figure A.41: Technical drawings for tube version, sheet 3
Figure A.42: Technical drawings for tube version, sheet 4
Figure A.43: Technical drawings for tube version, sheet 5
Figure A.44: Technical drawings for tube version, sheet 6
## Figure A.45: Technical drawings for tube version, sheet 7

<table>
<thead>
<tr>
<th>Elementnummer</th>
<th>Titel</th>
<th>Material</th>
<th>Menge</th>
<th>Kommentar</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Damptube</td>
<td>Cast Acrylic</td>
<td>1</td>
<td>McMaster-Carr PIN 8460K567</td>
</tr>
<tr>
<td>2</td>
<td>Cover</td>
<td>Delrin</td>
<td>1</td>
<td>McMaster-Carr PIN 6576K35</td>
</tr>
<tr>
<td>3</td>
<td>Cover</td>
<td>Delrin</td>
<td>1</td>
<td>McMaster-Carr PIN 6576K35</td>
</tr>
<tr>
<td>4</td>
<td>Mass</td>
<td>Brass</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>O-Ring</td>
<td>Rubber</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Rod</td>
<td>Stainless</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Nut</td>
<td>Iron</td>
<td>2</td>
<td>Century Spring Corp. PIN 11669</td>
</tr>
<tr>
<td>8</td>
<td>Washer</td>
<td>Iron</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Spring</td>
<td>Iron</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>O-Ring</td>
<td>Rubber</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Plug</td>
<td>brass</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NAME</th>
<th>DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hauschild</td>
<td>04/25/05</td>
</tr>
</tbody>
</table>

**SOLID EDGE**
EDS-PLM SOLUTIONS

**TITLE**
Damping mechanism for Whirl-I

**SCALE**
SHEET 7 OF 7
A.6 Measurements with and without the damper
Figure A.46: Measurement for spinrate -60°/sec

Figure A.47: Measurement for spinrate -70°/sec
Figure A.48: Measurement for spinrate $-75^\circ/\text{sec}$

Figure A.49: Measurement for spinrate $60^\circ/\text{sec}$
Figure A.50: Measurement for spinrate 70°/sec

Figure A.51: Measurement for spinrate 75°/sec
A.7 Comparison of damping fluids
Figure A.52: Comparison of damping fluids, spinrate $-60^\circ$/sec

Figure A.53: Comparison of damping fluids, spinrate $-70^\circ$/sec
Figure A.54: Comparison of damping fluids, spinrate -75°/sec

Figure A.55: Comparison of damping fluids, spinrate 60°/sec
Figure A.56: Comparison of damping fluids, spinrate 70°/sec

Figure A.57: Comparison of damping fluids, spinrate 75°/sec