# Far-Field Underwater Explosion (UNDEX) Fluid Modeling using Acoustic Elements

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Fluid-structure interaction in the far-field UNDEX problem is complicated by the shock wave traveling through the fluid, and by cavitation in the fluid. Numerical methods using acoustic formulations have frequently been applied to study far-field underwater explosion (UNDEX) effects on ships. This paper discusses the common assumptions made in acoustic formulations of far-field UNDEX problems, emphasizing the limitations of the method. A simple LS-DYNA model of a three-dimensional box barge subjected to a far-field underwater explosion is used to study the fluid model and the structural response. Alternative models and methods are discussed.

## Introduction and Background

The underwater explosion (UNDEX) and subsequent ship interaction problem has many unique characteristics all of which are governed by complex physics [1]. The complexity of the problem makes it extremely difficult to generate analytical solutions for the UNDEX problem except for the most simple of cases (eg., see [2]). While solutions to simplified problems provide a solid foundation for understanding the nature of UNDEX problems they do not predict more complex explosion and structure interactions encountered in real ship design problems.

Because of this complexity, predicting the effect of an UNDEX on real ships has been limited to using basic numerical methods. Some examples of these basic methods include using empirical relations obtained by experiment to predict UNDEX properties, the Shock Factor concept [1], and studying the ship response by applying a pressure load while neglecting the surrounding fluid. None of these methods have the capability to accurately model the entire UNDEX problem because they make many assumptions that limit their application to specific UNDEX characteristics or cases. However, with advances in computing technology and the progression of numerical methods, such as the finite element method, it has become possible to create more accurate and complete UNDEX models. With these models it may now be possible to predict UNDEX phenomena with sufficient accuracy for design applications.

## Far-Field UNDEX

There are two classes or types of underwater explosions currently being studied with numerical methods. In the first, the explosive charge detonates close to the ship hull. This is called a near-field underwater explosion. In the second, the charge is further away from the ship hull. This is called a far-field underwater explosion. The distance at which a problem becomes far-field is defined typically by the ship response.

In most cases, a charge that detonates close to the ship's hull causes the hull to rupture or causes large scale plastic deformation. As the charge is detonated further away from the hull, a point is reached where the hull no longer ruptures, but only plastic deformation occurs. Further increasing the charge distance reaches a point where only elastic deformation occurs. This is a far-field explosion. Traditionally the ship is assumed to be loaded as a whole, and the response is global [3,4]. However, in this work this assumption is not made. In this work the following assumptions define a far-field explosion: 1) the distance of the explosion is far enough away from the ship that neither the shock wave or bubble effects can rupture the hull, 2) a far-field explosion occurs far enough away from the ship that the bubble pulses damp out before reaching the ship structure; so the pulses do not affect the late time response of the ship, and 3) the far-field explosion occurs deep enough that the gas bubble does not migrate or the charge is close enough to the surface that the bubble vents before its first contraction.

## Far-Field UNDEX Models

Far-Field UNDEX models must typically treat two UNDEX phenomena: shock loading and cavitation. Shock loading occurs as a result of the incident explosive shock wave impinging on the wet-surface of the ship structure. Cavitation occurs because the pressure in the surrounding water drops below its vapor pressure due to the tensile wave reflected from the ship(local cavitation) or from the free surface (bulk cavitation).

The most common numerical method used to model far-field UNDEX is the finite element method (FEM). Considering shock loading and cavitation in FEM models creates inherent difficulties in creating an accurate model. Shock waves are characterized by a discontinuous rise in pressure followed by a brief period of exponential decay. Discontinuities can not be captured exactly in a FEM scheme, thus in the far-field problem distortion of the wave front and loss of pressure magnitude are two difficulties which must be overcome when modeling the explosive shock wave. Cavitation is a non-linear phenomena that requires specific treatment in the governing equations of the far-field model. Cavitaiton regions in FEM models are known to exhibit spurious oscillations known as “frothing” [6]. The accuracy of cavitation models may also be dependent on the ability of the numerical model to capture the shock wave, as the pressure of the reflected shock affects the spatial distribution of cavitation regions [7] which then effects the loading.

Far-Field UNDEX models that treat only shock loading may be solved by boundary element methods such as the DAA (see eg., [8]) in which only a structural model is needed. Such treatment eliminates the difficulties associated with numerically capturing the propagation of the shock wave, as it simply applies the shock to the wet surface of the structure. However, in recent years researchers have shown the importance of cavitation effects on the structural response [7,9,10]. When cavitation effects are included in the model the region of fluid surrounding the structure must be modeled and coupled to the structural model [11]. Such treatment means the difficulties associated with modeling the shock wave and cavitation must be resolved.

A typical FEM model of a far-field UNDEX seen in Fig. 1 consists of three parts, the structural domain, the fluid domain, and the fluid boundary. The focus of this work is on the numerical treatment of the fluid domain, although some attention is given to the fluid boundary which is typically a non-reflecting boundary.

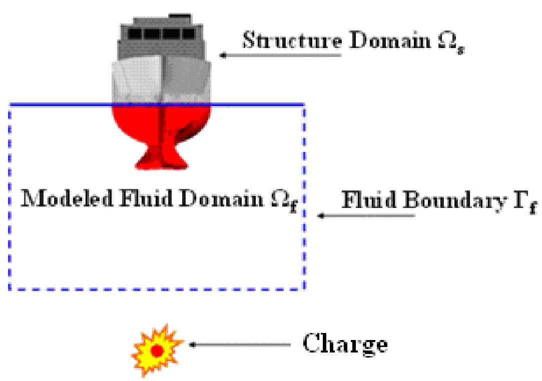


Figure 1. Schematic of a typical far-field UNDEX numerical model that includes cavitation effects

## Acoustic Elements

The most common approach to treating the fluid domain in a far-field UNDEX FEM model is to treat the fluid as an acoustic fluid. An acoustic fluid is one in which disturbances in the fluid propagate rapidly and are linearizable, even if the constitutive behavior of the fluid is non-linear [12]. Such a treatment is valid in the far-field problem, because shock and cavitaiton loading are early time events, occurring on the order of micro-seconds. Because of the short time duration of these events, the ship as whole does not exhibit a global response to the loading. Thus the far-field problem is of short duration and the global fluid displacement is small, which are two conditions for acoustic treatment given in [13].

The governing equation of an acoustic fluid can be formulated in two ways: a vector formulation, or a scalar formulation. The preferred choice for far-field UNDEX models is a scalar formulation because typical far-field UNDEX models require large amounts of fluid to be modeled. A scalar formulation reduces the number of unknowns at a node in the fluid from 3 to 1 [12], thus saving computational costs in large fluid models. A scalar formulation also has the advantage of automatically enforcing the irrotationality of the fluid [12], as commonly done in fluid formulations for shock capture [14].

There are three widely used choices of dependent variables for the scalar formulation of the fluid. Formulations exist for pressure [13], velocity potential [15], and displacement potential [6]. Sandberg [16] has also introduced a scalar formulation based on the density of the fluid. For problems without cavitation, the choice of dependent variable is personal preference. Although, some researchers have found that use of the pressure formulation for fluid-structure interactions with shock loading is problematic because a discontinuity in pressure creates a singularity in the acceleration of the structure [15]. However, ABAQUS reports to have successfully used a pressure formulation in fluid-structure interaction problems with shock loading [17].

Newton [6,18] developed an early FEM cavitation model for a scalar formulation of an acoustic fluid. Newton's model uses a bilinear constitutive equation to treat the cavitation region as a homogenous region of total pressure equal to the vapor pressure of water (commonly taken as zero) [11]. This treatment makes is reasonable the cavitation regions in a far-field UNDEX can be classified as acoustic vaporous cavitation. In this type of cavitation the formation of a region of vapor filled bubbles results from a change in pressure (e.g., from the reflected shock wave) [19]. Thus, treatment of the cavitation region as a homogenous region at a total pressure equal to vapor pressure makes sense if the influence of bubble formation and migration can be ignored. Treating cavitation in this manner contains the non-linearity in the constitutive behavior of the fluid. Liu, *et.al.* [20] contend that treatment of a cavitation region in this fashion could result in inaccurate flow physics, however because we are not necessarily concerned with the hydrodynamics of the fluid in the far-field problem this is not a concern. A limitation that is of concern is the fact that by treating cavitation as a constant pressure region waves cannot propagate through the cavitation region [20]. This could be of concern in any problem where there is a possibility of a reflected shock wave interacting with a region of cavitated fluid.

Newton selected the displacement potential formulation [18] for the dependent variable. The displacement potential formulation was also used by Zienkiewicz, Paul, and Hinton [21] to study cavitation effects on dam loading. In their model an unconditionally stable implicit time integration scheme was introduced, with an iterative check for cavitation. The displacement potential formulation was latter used by Felippa and DeRuntz [11,12] in the development of the Cavitating Fluid Analyzer (CFA) code. The CFA code extended Newton's model to three dimensions, incorporated a conditionally stable explicit time integration scheme, a node-by-node (non-iterative) check for cavitation, and the fluid boundary coupled to the boundary element code USA for a non-reflecting boundary. This approach has been incorporated into the commercial FEM code LS-DYNA [22,23] and has been used by many researchers in far-field UNDEX modeling applications [7,9,22,24]. The common name for the approach used in the CFA code is now called the cavitating acoustic finite element (CAFE) approach. A recent development in the CAFE approach has been the application of spectral elements and field separation. This approach was introduced by Sprague and Geers [10,23] as an improvement to shock capture (field separation) and cavitation modeling (spectral elements) in the current CAFE approach. They called their new approach cavitating acoustic spectral elements, or CASE.

Another approach to modeling cavitation in the far-field UNDEX problem is the use of density as the dependent variable. The purpose of this approach is to treat the non-linear relation between density and the speed of sound. The density approach was proposed by Sandberg [16]. It is similar to the approach in [21] in that an iterative scheme must be used to check for cavitation.

More recently ABAQUS has developed a cavitating fluid model using pressure as the dependent variable. Like the CASE approach, ABAQUS uses of field separation for non-cavitation problems but does not apply field separation to problems that consider cavitation.

In this paper we apply the CAFE approach to a typical box barge far-field UNDEX problem (eg., see [7,10]) using LS-DYNA to asses the current capability of the approach to model a large scale 3-D domain.

## CAFE Formulation

In LS-DYNA the CAFE approach is implemented using the MAT_90 material (see [25]) in conjunction with 8-node solid acoustic elements. LS-DYNA refers to these elements as acoustic pressure elements. The governing equations for the CAFE approach are derived in detail in [11]. Here we present a brief derivation to highlight the assumptions made in the fluid model. To begin, it is assumed the fluid domain can be modeled as a Navier-Stokes fluid (Eq. 1)

$$\rho\left(\frac{\partial v}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=\rho b-\frac{\partial \sigma_{ij}}{\partial x_j} \tag{1}$$

$$\sigma_{ij}=-p\delta_{ij}+\lambda\frac{\partial v_k}{\partial x_k}\delta_{ij}+\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right) \tag{2}$$

In Eq. 1, $v$ is the fluid velocity, σ the stress in the fluid given by Eq. 2, $b$ the body forces in the fluid, ρ the fluid density and μ is the viscosity. In Eq. 2, $p$ is the total pressure in the fluid and λ is the fluid viscosity. When the flow is assumed to be inviscid the state of stress in the fluid, Eq. 2, can be described as the total pressure in the fluid. We also assume the fluid to have small compressibility ($\Delta\rho << \rho_o$) and neglect the convective acceleration term in Eq. 1. These assumptions reduce Eq. 1 to the equation of motion for an acoustic fluid :

$$\ddot{d}=-\frac{1}{\rho}\nabla p \tag{3}$$

where $d$ is the fluid displacement and $p$ is the total pressure. When the continuity equation, Eq. 4, is combined with the first time derivative of the linear equation of state, Eq. 5, the constitutive equation for total pressure and displacement, given in Eq. 6, is obtained.

$$\frac{\partial \rho}{\partial t}=-\rho\frac{\partial \dot{d}}{\partial x} \tag{4}$$

$$p=c^2\rho \tag{5}$$

$$\nabla d=-\frac{1}{K}p \tag{6}$$

In Eqs. 5 and 6, $K$ is the bulk modulus of the fluid and $c$ is the speed of sound in the fluid. Equations 5 and 6 can be combined to yield Eq. 7. The governing equation for a displacement potential element, Eq. 9, is derived by replacing the displacement vector in Eq. 7 with the displacement potential defined in Eq. 8,.

$$\nabla^2 d-\frac{1}{c^2}\ddot{d}=0 \tag{7}$$

$$\rho d=-\nabla\psi \tag{8}$$

$$\nabla^2\psi-\frac{1}{c^2}\ddot{\psi}=0 \tag{9}$$

To account for cavitation the condensation, $s$, defined in Eq. 10 is used to modify the linear equation of state, Eq. 5, to a bilinear equation of state given in Eq. 11. Equation 11 provides a check for cavitation at each node that sets the total pressure in the fluid equal to zero if cavitation is present. Physically this can be interpreted as the fluid not being able to transmit negative pressures. Using the bilinear equation of state the governing equation for a displacement potential element, Eq. 9, is modified to its final form shown in Eq. 12.

$$s = \nabla^2 \psi \tag{10}$$

$$p = \begin{cases} p_{hydrostatic} + c^2 s, & s > -p_{hydrostatic} / c^2 \\ 0, & otherwise \end{cases} \tag{11}$$

$$\ddot{\psi} = \begin{cases} p - p_{hydrostatic}, & s > -p_{hydrostatic} / c^2 \\ -p_{hydrostatic}, & otherwise \end{cases} \tag{12}$$

The assumptions made in the fluid to derive the displacement potential formulation have potential implications on the fluid model in the box barge UNDEX model. By neglecting viscosity we are not accounting for interactions that involve the boundary layer such as separation and subsequent slamming that may occur between the fluid and the structure. We are also neglecting the motion of the ship due to buoyancy (i.e., heave, pitch, roll, etc.). Although these effects may not be important in current far-field UNDEX problems, it is important to understand the limitations of the CAFE approach.

## Box Barge Model

The far-field UNDEX problem described in this paper is a box barge ship structure subjected to a 20 lb. TNT explosion. The charge is placed 18 meters below the keel at amidships of the box barge as shown in Fig. 2. The fluid is modeled using the MAT_90 material model and 8-node solid acoustic elements. Standard salt water properties, $\rho$=1025 kg/m$^3$ and c=1514 m/s, are used for the fluid in the model. The dimensions, given in Fig. 3, of the fluid model are 15m x 15m x 7 m. The depth of the fluid mesh (7 m) is extended well beyond the lower boundary of the bulk cavitation region as required in CAFE modeling [11]. The fluid is modeled at the fluid-structure boundary with all nodes coincident. This is a requirement for treatment of the boundary interaction terms at the fluid-structure interface in the CAFE approach [11,9,23]. In LS-DYNA the incident shock wave is initialized at a node one element away from the structure using the peak pressure approximation [26]. This eliminates some difficulty in capturing the shock wave propagation, as it only needs to travel through one element before reaching the structure. This reduces potential distortion and dissipation of the wave as may occur when applied at the bottom boundary of the fluid.

The bottom boundary of the fluid is treated using the viscous boundary condition (VBC) in LS-DYNA. The VBC [27] is a non-reflecting boundary that acts as a numerical damper. We use the VBC instead of using the DAA via LS-DYNA/USA coupling as done in [7,9,11] to save computational time. The VBC does not treat the added mass effects of the fluid, in the DAA so results obtained with the VBC differ slightly from results obtained with the DAA. The size of the fluid model in this work is larger than a typical LS-DYNA/USA fluid model so that added mass effects are contained within the modeled fluid. We suggest that for simple problems, extending the fluid mesh is less computationally expensive than using the DAA.

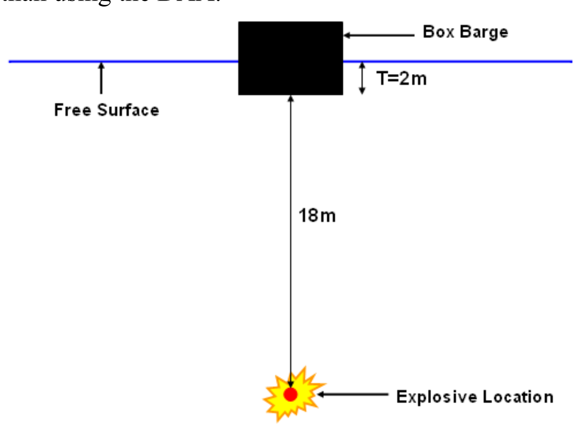


Figure 2. Schematic of box barge problem setup

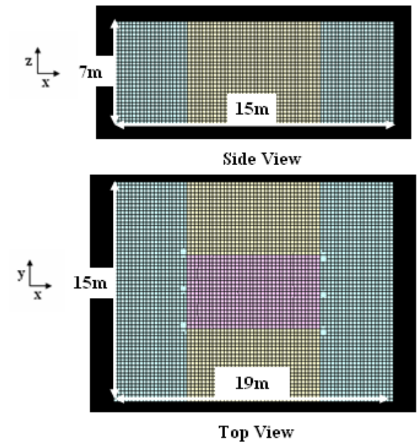


Figure 3. Side and top view showing dimensions of the 25 cm fluid mesh

To analyze the effects of the fluid model on the results of the box barge UNDEX simulation four different fluid meshes were created. The first two fluid meshes are a coarse mesh of 50cm elements, and a medium mesh of 25cm elements. The third fluid model is a fine mesh that uses 15cm elements. Table I gives the total number of elements used in each fluid model.

Table I. Number of total elements in the box barge model for each fluid mesh

| Mesh Size | Elements |
|---|---|
| **50 cm** | 16595 |
| **25 cm** | 135731 |
| **15 cm** | 264097 |

The box barge structural model is shown in Fig. 4. It is 9 meters in length with a constant beam and depth of 5 meters. Other dimensions are given in Table II. The structure of the box barge consists of the bottom, inner bottom, longitudinal girder, two traverse frames, a deck, sides, stern and bow. An internal view, showing the layout of the box barge structure is shown in Fig. 4. The box barge was modeled using Belyschko-Tsay shell elements with the MAT_ELASTIC material model. The MAT_ELASTIC material model allows only elastic deformation. The material properties used for the box barge model are given in Table III. The light ship draft of the box barge is 1.044 meters but to make modeling the waterline easier three equally distributed mass nodes were added along the centerline of the bottom hull of the box barge to make the draft 2 meters.

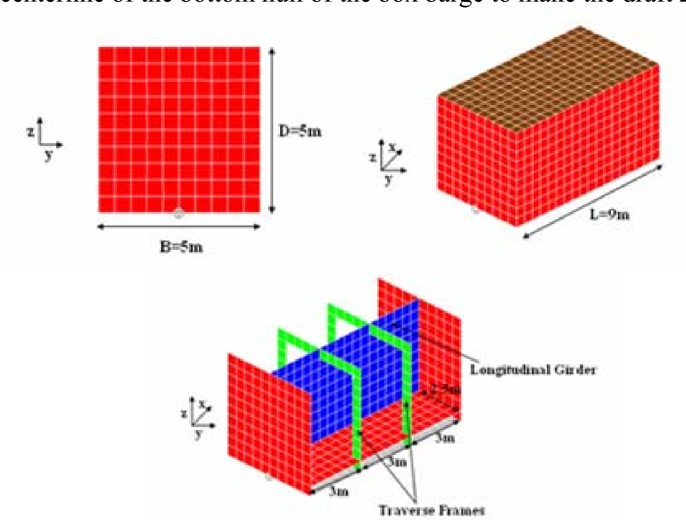


Figure 4. Front and isometric view of box barge showing beam, depth, and length (50cm mesh)

Table II. Box barge geometry

| | |
|---|---|
| **Length** | 9 m |
| **Beam** | 5 m |
| **Depth** | 5 m |
| **Draft (light ship)** | 1.044 m |
| **Displacement (light ship)** | 48174 kg |

Table III. Material properties used for box barge model

| | |
|---|---|
| **Density** | 7780 kg/m$^3$ |
| **Young's Modulus** | 1.9x10$^{11}$ Pa |
| **Poisson's Ratio** | 0.281 |

## Results

To assess the box barge model, we consider the vertical velocity response at a node on the keel of the box barge and the fluid pressure at an element on the fluid-structure interface. The vertical velocity response is considered because it dominates the box barge response for the modeled UNDEX scenario. We compare LS-DYNA/VBC results without cavitation to results obtained with LS-DYNA/USA (also without cavitation) and then compare the LS-DYNA/VBC results without cavitation to the LS-DYNA/VBC results with cavitation to determine the effect of cavitation on the structural response. Figure 5 compares the LS-DYNA/VBC without cavitation results for the vertical velocity at a point on the bottom keel obtained with the three mesh sizes compared to a solution obtained with the boundary element code USA. In USA the velocity rises to a maximum velocity of ~3 m/s almost instantaneously. Following this there is a sharp drop in velocity, followed by a second rise, and then a final sharp drop to ~ -1.5 m/s. After the drop to -1.5 m/s, the USA solution exhibits a gradual oscillatory motion. In Fig. 5 it is seen that although none of the LS-DYNA solutions match the USA solution exactly. The largest difference is in the maximum velocity reached; only the 15cm LS-DYNA results reach a maximum velocity of 3 m/s showing significant mesh refinement is needed to match the USA result. The other difference is in the behavior immediately after the maximum velocity is reached, in LS-DYNA only the 50cm results show a significant drop below 0 m/s as seen in the USA results. After the initial result the LS-DYNA results, like the USA results exhibit a gradual oscillatory response.

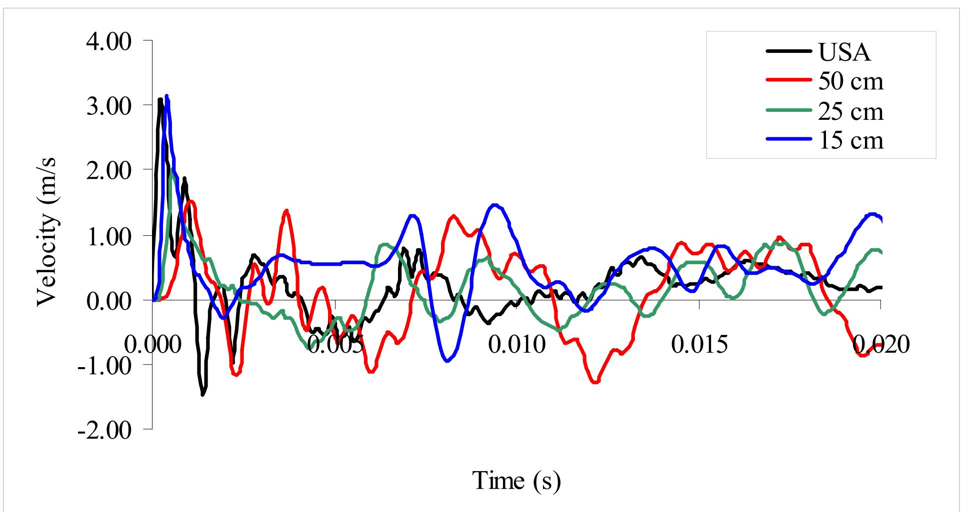


Figure 5. USA & LS-DYNA result comparison for box barge keel velocity response without cavitation

Figure 6 shows the vertical velocity of the same keel location when caivtation is considered in the model. There is a large difference between the results in Fig. 5 and Fig. 6. Figure 7 shows this more clearly as it compares results with and without cavitation. The results in Figs. 6 and 7, show the impact of cavitaiton on the structural response.

For each mesh size the rise to maximum velocity is the same regardless of cavitation effects. However, the drop in velocity from maximum is more gradual for each mesh size. The caviation effect is the greatest in the 15cm mesh results shown in Fig. 7, as the time taken for the velocity to go from 3 m/s to 0 m/s is greatly increased when cavitation is considered. A sharp spike in velocity is also seen at 0.017 s in the 15cm results. This spike is not seen in either the 50cm or 25cm mesh results.

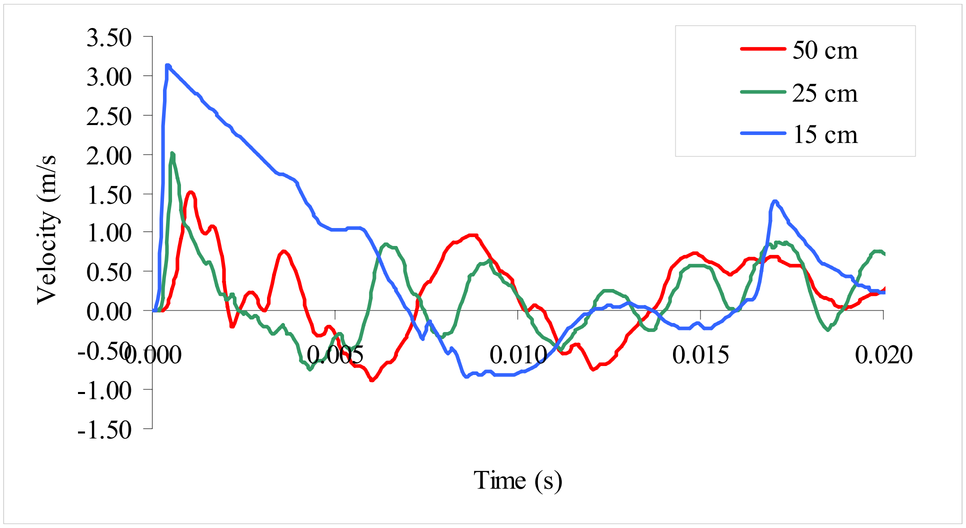


Figure 6. LS-DYNA result comparison for box barge keel velocity response with cavitation

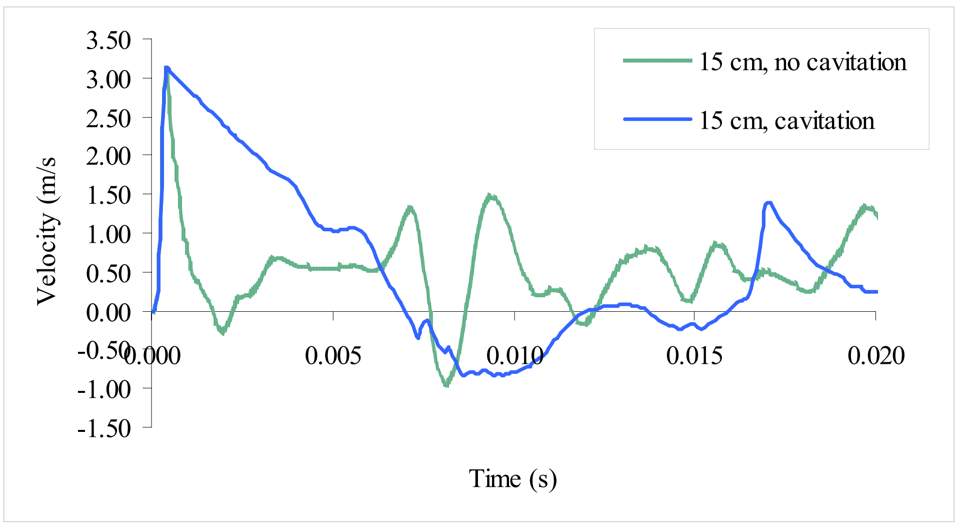


Figure 7. Comparison of results obtained with and without cavitation for the 25 cm and 15 cm fluid meshes

To better understand the difference in velocity responses seen in Figs. 5 and 6 we look at the pressure time history of an element at the center of the hull on the fluid-structure interface. Figure 8 shows the pressure time history for the hull element when cavitation is not considered compared to the peak pressure approximation [26]. Refining the mesh allows the incident shock wave to be captured with increasing accuracy. From these results we estimate that in order to capture the actual peak pressure the fluid mesh in the box barge model must be refined to an element size of ~5 cm. Because the 15cm model captures the peak pressure the best it also captures cavitation more accurately than the other models. This is shown in Fig. 9 where the cavitation region at the fluid-structure interface in the 15cm mesh remains cavitated longer than in the two coarser meshes. The closure pulse in the pressure time history at 0.017 s for the 15cm causes the velocity spike seen in Fig. 7 at nearly the same time. The 15cm model is the only model to exhibit such a response to the closure pulse. A similar pulse at 0.012 s for the 25cm mesh seems to cause no change in the velocity response (Fig. 6). Thus, we conclude that because the 15cm mesh captures the peak pressure and cavitation region, the structural results using the 15cm mesh are most reliable. This conclusion implies that even if the shock wave is initialized one element away from the structure, shock capture is still critical to accurate far-field UNDEX models.

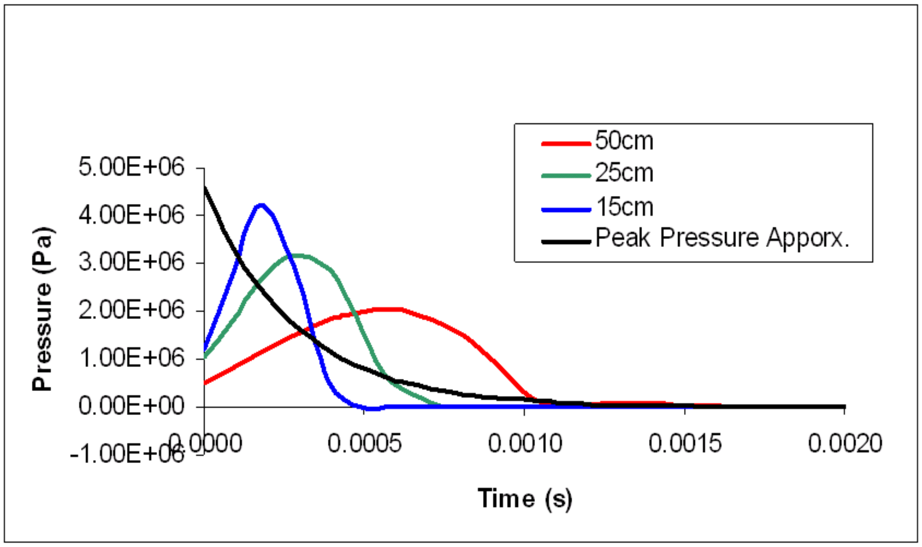


Figure 8. Comparison of pressure-time histories at fluid-structure interface for different LS-DYNA fluid models showing ability to capture shock wave with no cavitaiton

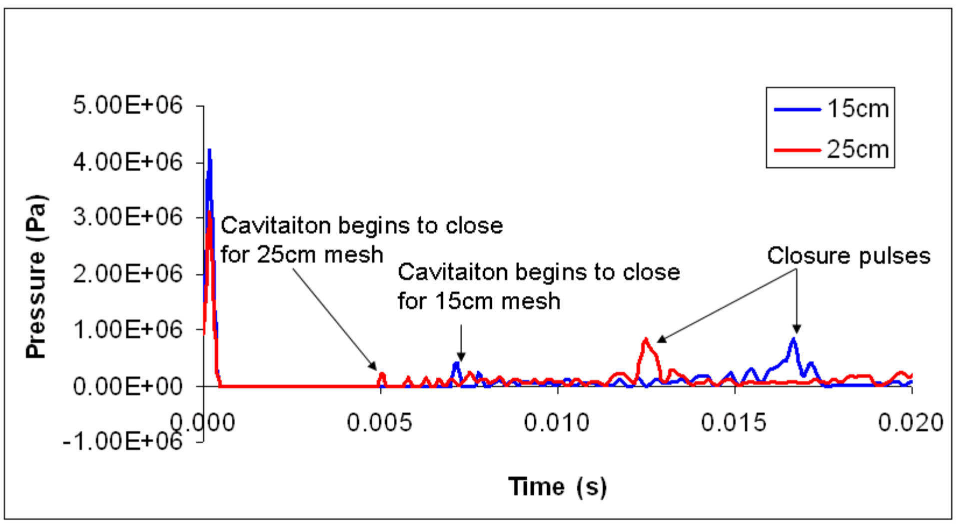


Figure 9. Comparison of pressure-time histories at fluid-structure interface for different LS-DYNA fluid models with cavitaiton

**Assessment of CAFE Elements**

The results of the previous section imply that shock capture plays an essential role in the accuracy of CAFE far-field UNDEX models. This contradicts previous thinking on shock wave modeling in the UNDEX problem. Past works [11,12] have assumed that it is sufficient for models to capture the correct impulse of the shock wave. When the impulse was correct, capturing the exact magnitude of the shock wave was not considered important because it was assumed that the structural response was driven by impulse and not peak pressure [4]. Although the initial structural response due to the shock wave may be driven by impulse, the 15cm model has shown the peak pressure of the shock wave must be captured correctly for an accurate model of cavitation and structure reloading. It is also known that the time duration of the incident pressure loading has an effect on the local cavitation [26]. Thus capturing the discontinuous rise and exponential decay of the shock wave is of importance.

The original CAFE models [11,12] do not consider the free surface of the fluid domain. Bulk cavitation was not treated in these models. Not only must the incident shock wave propagate to the structure in the box barge problem, it must also propagate to the free surface, a distance of 2 m. While the shock wave may not be distorted as it travels to the structure one element away, clearly there is the potential for distortion of the shock wave as it propagates to the free surface even for refined meshes. If the shock wave loses magnitude as it moves toward the free surface, then as the results of the previous section show (Figs. 8 and 9), the accuracy of the bulk cavitaiton region is affected. These findings are supported by Sprague and Geers who cite the need for highly refined CAFE meshes for accurate cavitation results [28]. Mesh refinement is undesirable because it increases the computational cost. This is especially true in large scale models where the scale of the model itself presents computational difficulties. Sprague and Geers [28] also point out that any mesh refinement in the fluid must also be carried out in the structure to maintain coincident nodes on the fluid structure interface. They propose the use of spectral elements as a means of more effectively refining the mesh and use a coupling algorithm on the fluid-structure interface to eliminate the need for coincident nodes [23].

Although spectral elements have been shown to provide some solutions to the deficiencies in the CAFE approach, it is our recommendation that the use of a numerical shock capturing scheme for the far-field UNDEX problem be investigated because of the potential large propagation distances of the shock wave to the free surface of the fluid.

## Conclusions

We have discussed the formulations commonly used in modeling far-field UNDEX explosions with numerical methods. To test the capabilities of current acoustic elements, the displacement potential formulation is applied to a box barge far-field UNDEX problem using LS-DYNA. Through the results of this model it is found that in order to accurately capture the explosive shock wave a highly refined mesh is required. Furthermore it is shown that accurately capturing the shock plays an important role in capturing the maximum velocity and cavitation response of the structure. It is proposed that shock capture and thus cavitation capture may be improved by the inclusion of a numerical shock capturing scheme in current acoustic formulations of the far-field UNDEX problem.

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