Assessment of Non-Reflecting Boundary Conditions for Application in Far-Field UNDEX Finite Element Models

Bradley Klenow and Dr. Alan Brown
Department of Aerospace and Ocean Engineering
Virginia Polytechnic Institute and State University
215 Randolph Hall
Blacksburg, VA 24061

Effective boundary conditions are important in UNDEX models because of the large physical extents of their relevant computational domains, and the need to limit these domains for computationally feasible solutions. In the far-field UNDEX problem it is common to model external fluid boundaries with non-reflecting boundaries. This paper assesses non-reflecting boundary conditions for application to far-field UNDEX. Two far-field UNDEX problems are considered: a three-dimensional plate and a three-dimensional box barge. Both are infinite domain problems which are treated with LS-DYNA's viscous boundary condition and the DAA boundary. The results from each boundary condition are compared. Other solutions are suggested.

Introduction and Background

The underwater explosion (UNDEX) and subsequent ship interaction is a problem that is of considerable interest in naval ship research. A large part of this research has addressed numerical models of ship shock trials. Ship shock trials are performed by detonating explosive charges at various locations around the ship and recording the ship response. Numerical models of shock trials are desirable due to the high cost and safety/environmental concerns associated with actual shock trials, which limits their application in the ship design process.

In most ship shock trials the UNDEX that occurs can be classified as a far-field UNDEX. A far-field UNDEX is one in which only elastic deformation occurs in the ship structure and the effects of the explosive gas bubble are neglected. Because the effects of the gas bubble are neglected, numerical models of far-field explosions only consider the phenomena of shock loading and cavitation. Shock loading and cavitation are part of the early time response of the ship. They occur in microseconds. Late time response effects are loading from bubble pulses and the gas bubble itself, which are not typically considered in the far-field problem.

Numerical models that only consider shock wave loading do not require the fluid surrounding the ship to be modeled in the computational domain. However, when the effects of cavitation are considered the surrounding fluid must be modeled [1]. The most frequently used method for numerical treatment of the surrounding fluid is the finite element method (FEM). This creates a difficulty because mathematically the fluid region should be treated as an infinite domain because physically it is part of a large body of water. However, the fluid domain in the FEM model is finite and thus will not behave the same as the infinite fluid in the physical problem. An obvious solution to this problem is to create a very large FEM model such that the finite boundary has no effect on the results. However, this is impractical, especially in far-field UNDEX models where the computational domain can be very large, due to large amount of computation time needed. A more practical and efficient solution to finite element (FE) consideration of infinite domains is the use of non-reflecting boundary conditions.

Non-reflecting Boundary Conditions

Non-Reflecting boundaries (NRBCs) are used to model a problem with an infinite domain using a finite model. The term non-reflecting is synonymous with the terms silent, transmitting, absorbing, and radiating. We use the term non-reflecting here because in a wave propagation problem we seek to eliminate reflections from the boundary. Many types of NRBCs are available, but in general they all act to prevent energy radiating toward infinity from being reflected back into the model at the finite boundary of the model. An excellent introduction to the subject of non-reflecting boundaries can be found in [2].

Some common types of non-reflecting boundaries in FE domains are: coupled finite element-boundary element methods (FE-BE), artificial boundary conditions, filtering schemes, and infinite elements. These methods can be divided into those that are approximate, meaning they approximately treat the infinite domain behavior, or exact, meaning they exactly treat the infinite domain behavior.

Coupled FE-BE methods are approximate non-reflecting boundaries that use the FEM in the finite domain of the problem and the boundary element method (BEM) applied at the boundary of the finite domain to treat the infinite domain. Such methods are useful because the properties of the infinite domain are treated with the BEM, allowing effects like the added mass of the fluid to be accounted for in the model. The main disadvantage to this method is the higher computational cost compared to other boundary conditions, such as artificial boundary conditions. An example of a coupled FE-BE non-reflecting boundary is the use of the LS-DYNA/USA coupling in far-field UNDEX models [3,4].

Artificial boundary conditions are applied via a boundary term and can be exact or approximate non-reflecting boundary conditions. There are two classifications of artificial boundary conditions, local and non-local [2]. Local artificial boundary conditions are computationally simple but only approximate the infinite domain. They can be exact but only in the most simple of problems. In most cases the accuracy of local artificial boundary conditions accuracy depends heavily on the type of problem they are used for and the size of the computational domain [2]. An example of a local artificial boundary condition is the Higdon NRBC, which acts as a numerical damper to absorb incident waves [5,6].

Non-local artificial boundary conditions are exact non-reflecting boundary conditions for a much greater range of problems than local artificial boundary conditions. However, non-local artificial boundary conditions are much more complex and require a much greater amount of computation in their derivation. An example of a non-local artificial boundary condition is the Dirichlet-to-Neumann (DtN) condition discussed in [2].

Filtering schemes are approximate NRBCs that use an artificial material that is applied to the boundary of the domain. The artificial material extends the boundary of the domain but uses varying material properties to filter/damp incident waves before they reach the boundary of the artificial material. Examples of filtering schemes can be found in [7,8]. The main drawback to filtering schemes is that the computational domain of the problem must be extended.

Infinite elements are another method of imposing a NRBC in a FEM model. Infinite elements are approximate NRBCs that use special elements at a boundary to create a NRBC. The concept of infinite elements is to use finite elements to define a semi-infinite domain at the boundary which is desired to be a NRBC. These elements are integrated in the model using the same methodology as regular finite elements. The difference is that the infinite elements use special basis functions that are defined over the infinite domain of the element rather than the standard finite element basis functions. Defining the basis functions in this manner allows the infinite element to copy the behavior of the exact solution of the problem at infinity [9]. Examples of infinite elements can be found in [10,11,12,14] and in the commercial FE code ABAQUS. The main advantage of infinite elements is that like coupled FE-BE methods, fluid behavior such as added mass effects can be treated. However, infinite elements offer the added benefit of being less computationally expensive than the BEM because they preserve the efficiency of standard FE formulations [13].

In this paper we model two far-field UNDEX problems using the FEM in order to assess the accuracy and computational efficiency of two NRBCs. The first NRBC assessed is the doubly asymptotic approximation (DAA) NRBC, which is a BE-FE coupling method. The second NRBC assessed is the viscous boundary condition (VBC), which is a local artificial NRBC.

Far-field UNDEX NRBCs

The two far-field UNDEX problems studied in this paper are the Bleich-Sandler plate and a three-dimensional box barge problem adapted from [15]. In both problems cavitation is considered, and therefore the fluid domain must be included in the models. To model the fluid domain in both problems the cavitating acoustic finite element (CAFE) is used. The CAFE approach was initially developed by Newton [16], but was expanded by Felippa and DeRuntz in [3]. CAFE elements treat the fluid as an acoustic domain with a bilinear equation of state that allows cavitation to be considered. The CAFE approach has been widely used in many far-field UNDEX problems [4,17,18].

Applying a NRBC to the CAFE fluid region in a far-field UNDEX FEM models requires several special considerations. The first is added mass effects. The global motion of the ship in the fluid is affected by the added mass of the fluid. Therefore in far-field UNDEX problems, it may be necessary for a NRBC to account for added mass effects or the fluid domain must be large enough such that all added mass effects are contained within the modeled fluid.

Another special consideration is the shape of the boundary. In most FEM models of far-field UNDEX, the interface conditions between the fluid and structure must have one-to-one nodal contact on the interface [4,15]. For complex three-dimensional geometries, such as naval ship structures, this creates the need for fluid elements to be orthogonal to structural elements on the fluid-structure interface [4]. The result of modeling the fluid in this way is that the outer boundary of the fluid is most often a curved surface. This can create problems for local artificial NRBCs that require rectangular boundaries and waves of normal incidence for optimal accuracy.

The most important special consideration in applying NRBCs to far-field UNDEX problems is cavitation. Because cavitation is a non-linear phenomena, and the NRBCs discussed above only treat linear phenomena, the maximum depth of the cavitation region must be entirely contained within the fluid domain of the model [3,4]. This can create computational difficulties in problems that consider bulk cavitation, as the depth of bulk cavitation regions can be large.

One of the first non-reflecting boundary used in the CAFE approach was the Somerfeld radiation condition [14,Z]. The Somerfeld radiation condition is a local artificial NRBC. It is exact for one-dimensional plane waves of normal incidence on rectangular boundaries, but it cannot be used effectively in far-field UNDEX problems with complex geometries. Sprague and Geers [18,19] have also used a local artificial NRBC in their far-field UNDEX models. They applied the curved wave approximation (CWA), which handles curved geometries better than the Somerfeld NRBC. The CWA is also implemented in the commercial FE code ABAQUS.

Felippa and DeRuntz [3] used a coupled FE-BE approach to treat the non-reflecting boundary in their CAFE program. The fluid domain was treated by their CAFE code, cavitating fluid analyzer (CFA), and the boundary element underwater shock analysis (USA) code was coupled to the CFA fluid boundary to treat the infinite domain. USA treats the fluid by applying the doubly asymptotic approximation (DAA) to the fluid boundary. The DAA is a system of differential equations in time that combines the plane wave approximation, which is exact at high frequency (early time response), with the virtual mass approximation, which is exact at low frequency (late time response) [20]. The DAA is beneficial because it treats added mass effects and can be used on a wide variety of boundaries geometries. However, it is computational expensive, especially for large scale far-field UNDEX models. Treatment of the NRBC using the DAA has been incorporated into the commercial FE code LS-DYNA via a special coupling option with USA [21]. Thus using the DAA as a NRBC is widely used in far-field UNDEX problems [4,15,17,22].

Because of the computational expense associated with using the DAA as a NRBC we seek to test the feasibility of using a local artificial NRBC rather than the more traditional DAA boundary. To accomplish this we use LS-DYNA to model both problems, using LS-DYNA/USA coupling for the DAA boundary and using LS-DYNA's own viscous boundary condition (VBC) as the local artificial NRBC.

The Viscous Boundary Condition

The viscous boundary condition (VBC) is a local approximate NRBC that was originally developed by Lysmer and Kuhlemeyer [23] for soil-structure interaction problems and later studied by Cohen and Jennings [24]. The viscous boundary condition calculates the normal and shear stress at each boundary node for an incident pressure wave, shown in Eq. 1. The incident stresses are matched at each node so that the resulting reflected stress is zero.

$$\sigma_{norm} = -\rho c_d v_{norm}$$
$$\tau_{shear} = -\rho c_s v_{\tan} \qquad (1)$$

In Eq.1 σ_{norm} is the normal stress at the boundary, τ_{shear} is the shear stress at the boundary, ρ is the mass density, c_d is the distortional wave speed, c_s is the dilatational wave speed, and v is the velocity.

In a CAFE fluid the changes in mass density, ρ, are assumed to be small. Therefore, only distortional waves can propagate in a CAFE fluid. This means only σ_{norm} need be matched when the VBC is used on a CAFE boundary. Viscosity is also neglected in a CAFE fluid, therefore the state of stress in a CAFE fluid can be described as the total pressure of the fluid, *p*. Therefore when the VBC is applied to a CAFE fluid mesh boundary it reduces to

$$p = -\rho c v_{norm} \tag{2}$$

where *c* is the speed of sound in the fluid.

The VBC treats waves of normal incidence almost exactly [23], but its accuracy degrades as the angle of incidence decreases from 90° to 0°. Also the VBC does not capture added mass effects, which makes it less accurate than the DAA for problems where added mass effects affect the ship response. However, Since Eq. 2 has the form of the plane wave approximation (PWA) the VBC and the DAA should behave similarly for the early time response of the CAFE model because both apply the PWA in the early time. Because we consider only early time phenomena in far-field UNDEX problems the VBC could offer significant savings in computational time over the DAA without greatly affecting the accuracy of results.

Bleich-Sandler Model

The Bleich-Sandler plate problem is a well known benchmark problem for far-field UNDEX models that consider cavitation [25]. The Bleich-Sandler problem was used to validate the CAFE approach [16] and has been solved using CAFE/DAA coupling in several studies [3,22]. It involves a flat plate that rests on the free surface of a 3.81 m fluid column. We model this problem in LS-DYNA using 8-node solid acoustic elements with the MAT_90 material model for the fluid domain and a single Belyschko-Tsay shell element with the MAT_ELASTIC material model for the flat plate. In the fluid domain 100 elements are used in the z-direction and 2 elements are used in the x and y-directions. This makes each element a square with 0.0381 m sides. All fluid and plate nodes are constrained so that only motion in the z-direction is allowed. This is required because the analytical solution to the problem used to validate the FE solution is derived in only one-dimension. Material properties of the fluid and plate are given in Table I. The geometry of the fluid and plate is described in Fig. 1 and Table I.

Table I. Material and geometry properties of the fluid and plate in the Bleich-Sandler problem

Fluid		**Structure**	
L	3.81 m	**l**	3.81 cm
w	3.81 cm	**w**	3.81 cm
t	3.81 cm	**ρ**	5696 kg/m^3
ρ	998 kg/m3	**E**	207 GPa
c	1450 m/s	**υ**	0.3

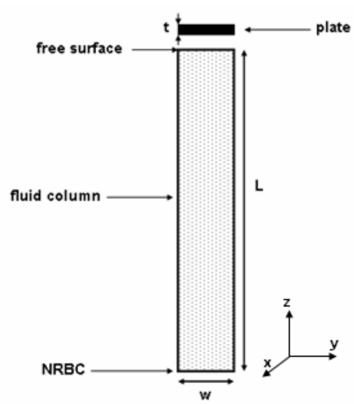

Figure 1. Schematic of the Bleich-Sandler plate problem

To create an UNDEX loading on the plate a plane incident shock wave with exponential decay is initialized one fluid element away from the structure. Properties of the shock wave are given in Table II. In Table II DP is the distance from the free surface to the charge.

Table II. Properties of the incident shock wave used in the Bleich-Sandler problem

DP	254000 m
$\mathbf{P_{peak}}$	710160 Pa
$\boldsymbol{\tau}$	996 ms

The bottom boundary of the fluid domain is treated using both the DAA via LS-DYNA/USA coupling and the VBC via LS-DYNA's BOUNDARY_NON_REFLECTING option. The vertical (z-direction) velocity of the plate is used to compare the results obtained with both methods The solution using the DAA is used as the benchmark solution because this method has previously been validated with the analytical solution of the problem [3,22]. Results obtained with using the VBC on the bottom boundary of the fluid are compared to results using the DAA in Fig. 2. The two solutions are virtually identical until the time reaches approximately 0.0125 s, at which point cavitation re-loading occurs and the two solutions begin to differ. It is known that the fluid mesh is of adequate length to be below the maximum depth of cavitation, and the boundary is not irregular in shape. Therefore, the small difference between the two solutions is caused by the fact that the DAA treats the added mass effects of the fluid and the VBC does not.

To demonstrate this several different lengths of fluid were tested. First the length of the fluid mesh was shortened by 25%, 50%, and 75%. Next the length of the fluid mesh was increased by 50%, 100%, and 200%. The lengths of all reduced and extended fluid meshes are given in Table III. Figure 3 shows that as the fluid mesh is truncated the results obtained using the VBC behave less and less like the results obtained with the DAA. The opposite of this effect is observed when the fluid mesh is extended, as shown by the L^2 error norms (where the DAA solution is taken as the analytical solution) of the results obtained with the extended fluid meshes given in Table IV. The L^2 norms show that the VBC solution converges when the fluid mesh is extended to about three times its original length. Although the VBC does not converge to the DAA solution, the error in between the VBC solution obtained with the largest fluid mesh and the DAA solution is small.

The results of the Bleich-Sandler problem show the potential for using an extended fluid mesh with the VBC instead of the DAA as a non-reflecting boundary condition in LS-DYNA far-field UNDEX models. In the next section we apply the VBC in a more complex far-field UNDEX model to see the potential computational savings of using the VBC in LS-DYNA.

Table III. Length fluid mesh for the increased and decreased fluid meshes

Increased	Length (m)
50%	5.715
100%	7.62
200%	11.43

Decreased	Length (m)
25%	2.8575
50%	1.905
75%	0.9525

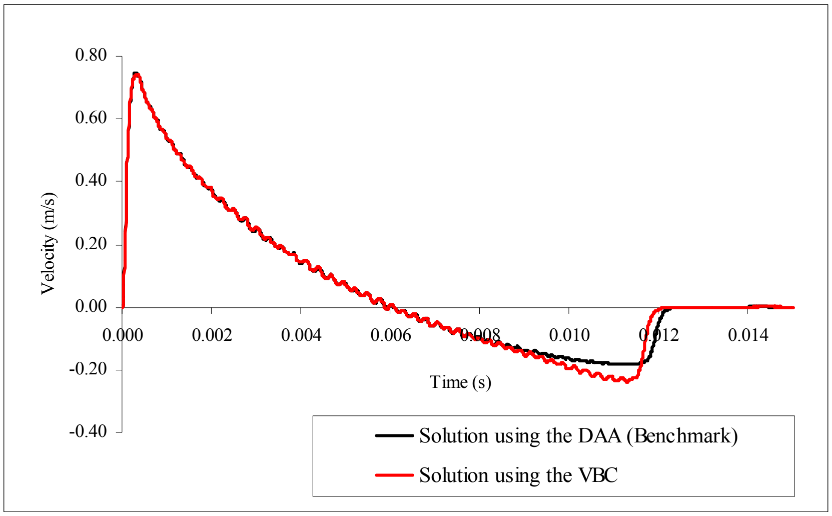

Figure 2. Vertical (z-direction) velocity of the plate using the DAA and VBC as fluid boundary conditions

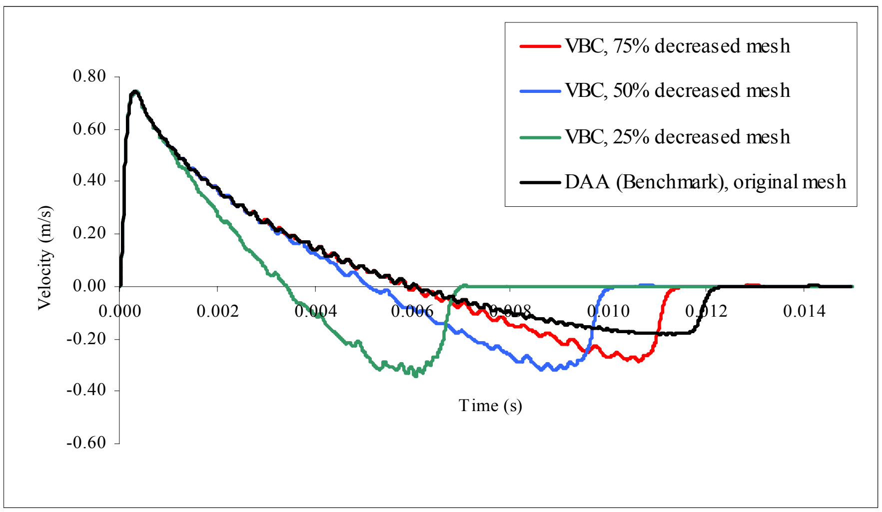

Figure 3. Effect of fluid mesh truncation on results obtained using the VBC as the fluid boundary condition

Table IV. L^2 error norms for the VBC solutions using extended fluid meshes

Fluid Length	L^2 Error Norm
VBC, original mesh	0.0870
VBC, 50% increased mesh	0.0159
VBC, 100% increased mesh	0.0158
VBC, 200% increased mesh	0.0158

Box Barge Model

The box barge UDNEX model is taken from [15], where it was solved using LS-DYNA/USA coupling. In the box barge problem a charge is detonated 4.724 m below the keel of the box barge, as shown in Fig. 4. Properties of the charge are given in Table V. The box barge structural model is shown in Fig. 5 and properties of the barge are given in Table VI. The box barge model consists of a stiffened internal structure, a keel, two transverse bulkheads, and four lumped masses to control the draft of the barge. The stiffeners and keel have rectangular cross sections and are modeled using beam elements. There are 10 evenly spaced transverse stiffeners along the length of the barge, 5 longitudinal stiffeners on the side hull, and 4 longitudinal stiffeners along the bottom hull with the keel placed along the centerline. The lumped masses are placed evenly along the keel. The Plating of the box barge is modeled using 4-node Belyschko-Tsay shell elements. LS-DYNA's MAT_ELASTIC material model is used for all box barge beam and shell elements.

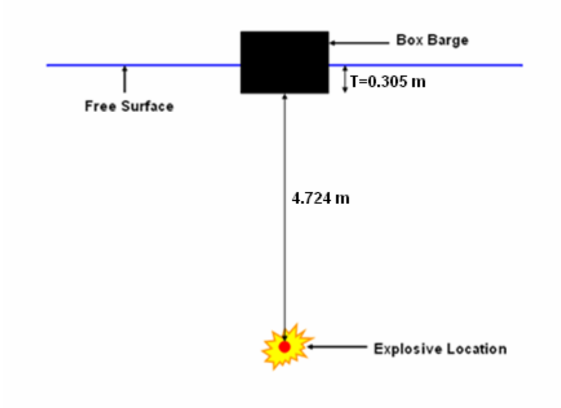

Figure 4. Schematic of the box barge problem

Table V. UNDEX parameters for charge used in box barge problem

P_{peak}	19.22 MPa
τ	0.2383 ms
DP	4.724 m

Table VI. Geometric and material properties of the box barge structural model

Length **Beam** **Depth** **Draft**	3.048 m 0.61 m 0.61 m 0.305 m
Stiffener height	0.051 m
Stiffener thick.	3.175 mm
Keel height	0.152 m
Keel thick.	6.35 mm
Lumped mass	1.475×10^6 kg/m^3

ρ	7861 kg/m^3
E	207 GPa
υ	0.3

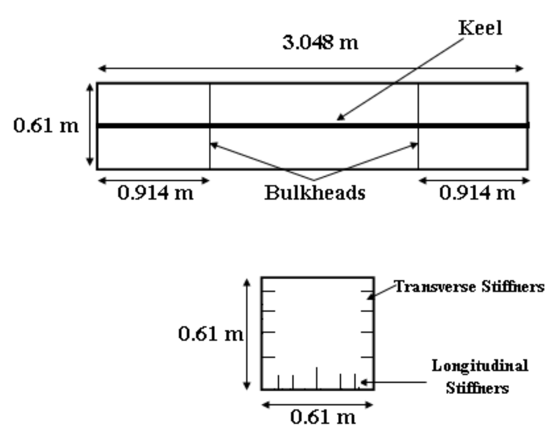

Figure 5. Top view and cross section of box barge structural model [15]

The geometry of the box barge fluid model is shown in Figure 6. It is modeled with 75344 8-node solid acoustic elements with the MAT_90 material model. Standard salt water properties are used for the fluid material properties. The depth of the fluid mesh is 3.56 m, which is twice the maximum depth of the cavitation region [15].

Cavitation is considered in the fluid model and the shock wave is initialized one element away from the box barge structure. Nodes on the fluid-structure interface are modeled to be coincident as required in CAFE fluid modeling [3,4]. All fluid boundaries except the free surface are treated with the DAA (via USA) or the VBC (via LS-DYNA).

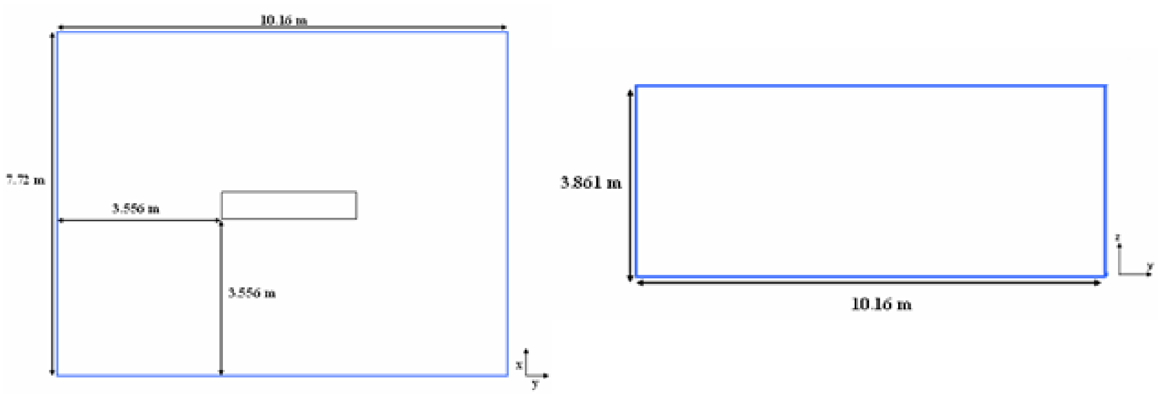

Figure 6. Side and top views of box barge fluid model showing dimensions

Figures 7 and 8 show the solutions obtained with the DAA and VBC for vertical velocity on a point on the keel of the box barge. As in the Bleich-Sandler problem, initially the two solutions are identical, but they begin to differ as the added mass of the fluid begins to affect the motion of the structure. The solution using the DAA took 1 hour and 42 minutes to run on a desktop PC, while the solution using the VBC took 10 minutes to run on the same PC. Thus, for large scale models the VBC offers a significant savings in computational time.

Because the fluid mesh used in the box barge problem is already twice as large as required, we infer from the Bleich-Sandler results that increasing the mesh size will not account for any more added mass effects when the VBC is used. Therefore we conclude using the VBC as a NRBC in LS-DYNA far-field UNDEX models offers significant savings in computational time over the DAA. However, there is a slight decrease in accuracy of the results due to the inability of the VBC to treat added mass effects even when a larger fluid mesh is used. A compromise between the DAA and the VBC is to use of the PWA in USA instead of the DAA. While the PWA and VBC have similar formulations (see Eq. 2), because the PWA is evaluated by the BE-FE coupling in USA it is inherently a more accurate treatment than the VBC which is a local artificial boundary condition [2]. In addition to increased accuracy over the VBC, computational time is still saved by using the PWA instead of the DAA in USA because the late time evaluation of the DAA is neglected. Figure 9 shows results for the box barge problem using the PWA in USA. The PWA solution is identical to the solution obtained with the DAA. The run time for the PWA solution was 16 minutes, so it is slightly more expensive than the VBC but much less expensive than the DAA and just as accurate.

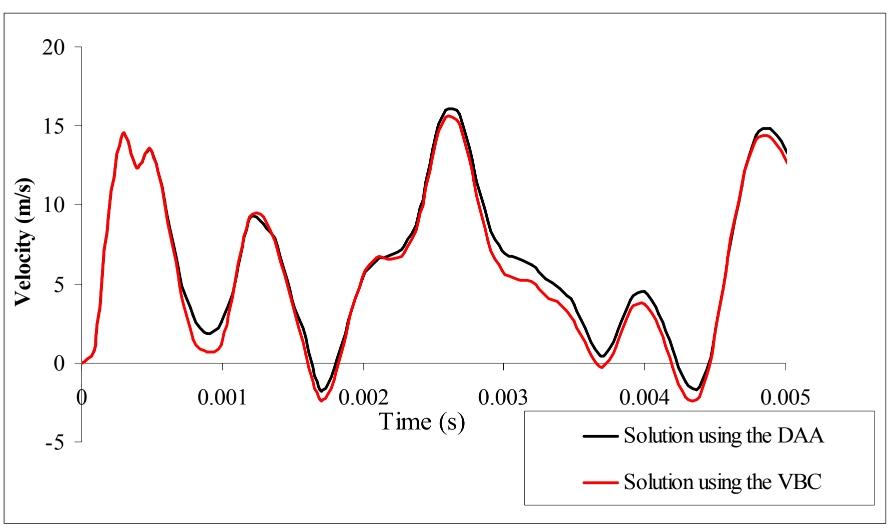

Figure 7. Comparison of DAA and VBC solutions for the vertical velocity at a point on the keel for the box barge problem (early time)

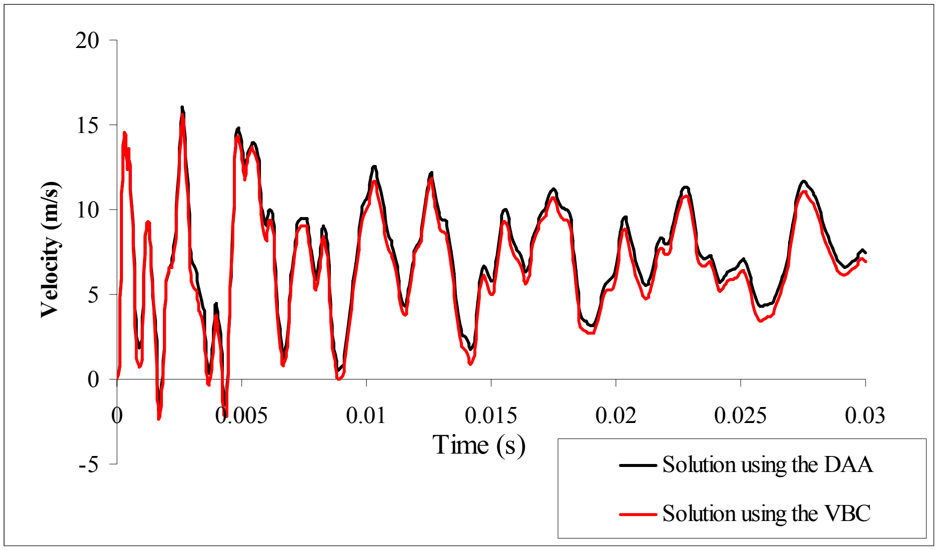

Figure 8. Comparison of DAA and VBC solutions for the vertical velocity at a point on the keel for the box barge problem

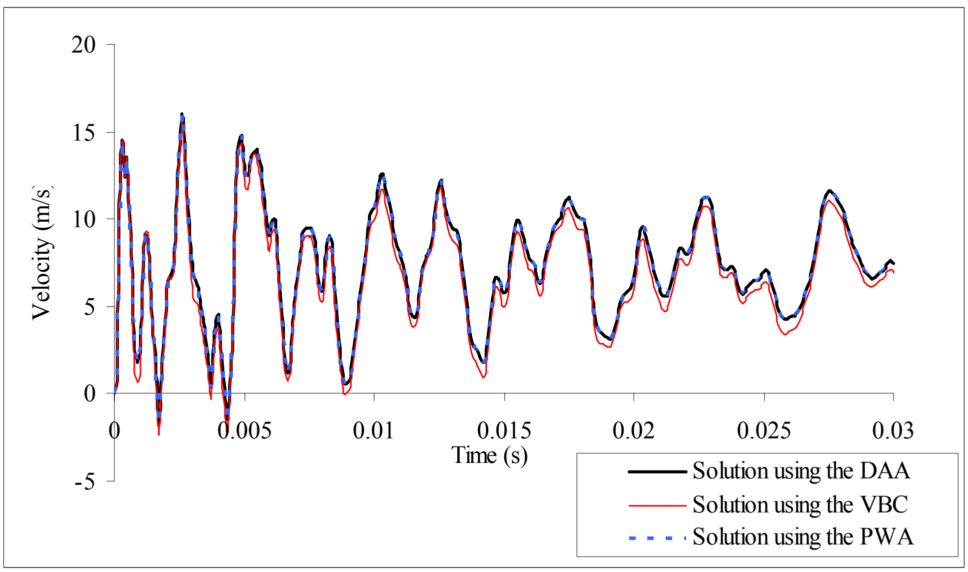

Figure 9. Comparison of DAA, VBC, and PWA solutions for the vertical velocity at a point on the keel for the box barge problem

Conclusions

We have studied the use of two non-reflecting boundary conditions for application to LS-DYNA models of far-field UNDEX problems. In the first problem, we solved the Bleich-Sandler plate problem using the traditional DAA boundary and compared it to solutions using LS-DYNA's viscous boundary condition. These results showed that if the fluid mesh is extended so that added mass effects are contained in the fluid mesh than solutions using the VBC compare well to solutions using the DAA. In the second problem we solved a large scale three-dimensional box barge problem. These results show that the VBC does not reproduce DAA results even with a significantly larger fluid mesh in a large scale problem. However, the difference between the two solutions is small and the VBC offers the advantage of significant computational savings. Using the PWA (via USA) as a NRBC offers computational savings over the DAA because it reproduces near-time DAA results exactly. Thus the PWA may be the most computationally efficient NRBC for use in LS-DYNA near-time far-field UNDEX models. Future work in this area includes testing the effectiveness of local artificial boundary conditions and BE-FE coupling methods on boundaries with complex geometries and non-normal incident waves. Other work is to test the use of infinite elements in far-field UNDEX problems and also the development of a NRBC capable of treating non-linear effects so that the fluid mesh boundary can be moved inside the lower boundary of the cavitation region.

References

1) Felippa, C.A. and DeRuntz, J.A., "Finite Element Analysis of Shock Induced Hull Cavitaiton," *Computational Methods in Applied Mechanical Engineering,* 1984, Vol. 44, pp.297-337.
2) Givoli, D., 1992, *Numerical Method for Problems in Infinite Domains*, Elsevier.
3) Felippa, C.A. and DeRuntz, J.A., "Acoustic Fluid Modeling by the Displacement Potential Formulation, with Emphasis on the Wedge Element," *Computers and* Structures, 1991, Vol. 41, pp.669-686.
4) Shin, Y. and Schneider, N. "Ship Shock Trial Simulation of USS Winston S. Churchill (DDG 81): Modeling and Simulation Strategy and Surrounding Fluid Volume Effects," *74th Shock and Vibration Symposium*, 2003.
5) Higdon, R.L., "Numerical Absorbing Boundary Conditions for the Wave Equation," *Mathematics of Computation*, 1987, Vol. 49, pp.65-90.
6) Higdon, R.L., "Absorbing Boundary Conditions for Acoustic Elastic Waves in Stratified Media," *Journal of Computational Physics,* 1992, Vol. 101, pp.386-418.
7) Israeli, M. and Orszag, S.A., "Approximation of Radiation Boundary Conditions," *Journal of Computational Physics*, 1981, Vol. 41, pp. 115-135.
8) Liu, G.R. and Quek Jerry, S.S., "A Non-Reflecting Boundary for Analyzing Wave Propagation using the Finite Element Method," *Finite Elements in Analysis and Design*, 2003, Vol. 39, pp. 403-417.
9) Kunar, R.R. and Rodriguez-Ovejero, L. "A Model with Non-Reflecting Boundaries for use in Explicit Soil-Structure Interaction Analyses," *Earthquake Engineering and Structural Dynamics*, 1980, Vol. 8, pp.361-374.
10) Zienkiewicz, O.C. and Bettess, P. "Infinite Elements in the Study of Fluid-Structure Interaction Problems," 1976, *Lecture Notes in Physics*, No.58, 133-172, Springer.
11) Bettess, P. and Zienkiewicz, O.C. "Diffraction and Refraction of Surface Waves Using Finite and Infinite Elements," *International Journal of Numerical Methods in Engineering*, 1977, Vol. 11, pp.1271-1290.
12) Zienkiewicz, O.C. and Bettess, P., Chiam, T.C., and Emson, C. "Numerical Methods for Unbounded Field Problems and a New Infinite Element Formulation," *Computational Methods for Infinite Domain Media-Structure Interaction*, 1981, AMD Vol. 46, pp.115-148, ASME.
13) Everstine, G.C., "Finite Element Formulations of Structural Acoustics Problems," *Computers & Structures*, 1997, Vol. 65, pp.207-321.
14) Cipolla, J.L., and Butler, M.J., "Infinite Elements in the Time Domain using a Prolate Spheroidal Multipole Expansion," *International Journal of Numerical Methods in Engineering*, 1998, Vol. 43, pp.889-908.
15) Wood, S.L., 1996, *Cavitation Effects on a Ship-Like Box Structure Subjected to an Underwater Explosion*, Master's Thesis, Naval Postgraduate School, Monterey, CA.
16) Newton, R.E. "Finite Element Analysis of Shock-Induced Cavitation," *ASCE Spring Convention*, 1980.
17) Gong, S.W., and Lam, K.Y., "On attenuation of floating structure response to underwater shock," *International Journal of Impact Engineering*, 2006, Vol. 32, pp. 1857-1877.
18) Sprague, M.A., and Geers, T.L., "A Spectral-Element/Finite-Element Analysis of a Ship-Like Structure Subjected to an Underwater Explosion," *Computer Methods in Applies Mechanics and Engineering*, Vol. 195, 2006, pp. 2149-2167.
19) Sprague, M.A., and Geers, T.L., "A Spectral-Element Method for Modeling Cavitation in Transient Fluid-Structure Interaction," *International Journal for Numerical Methods in Engineering*, 2004, Vol. 60, pp. 2467-2499.
20) Deruntz, J.A. "Application of the USA Code to Underwater Shock Problems," *73rd Shock and Vibration Symposium*, 2002.
21) LSTC, 2003, "LS-DYNA Keyword Users Manual" Version 970, Livermore Software Technology Corporation, Livermore, CA.
22) Shin, Y.S. and Santiago, L.D., "Surface Ship Shock Modeling and Simulation: Two-Dimensional Analysis," *Shock and Vibration*, 1998, Vol. 5, pp. 192-137.
23) Lysmer, J. and Kuhlemeyer, R.L. "Finite Dynamic Model for Infinite Media," *Journal of Engineering Mechanics*, 1969, ASCE, pp.857-877.
24) Choen M. and Jennings, P.C. "Silent Boundary Methods for Transient Analysis," in *Computational Methods for Transient Analysis*, 1983, North Holland, pp. 301-360.
25) Mair, H.U., "Benchmarks for submerged structure response to underwater explosions," *Shock and Vibration*, 1999, Vol. 6, pp.169-181.