VMCAv1.m User's Manual

A MATLAB m-File to Calculate the Single Engine Minimum Control Speed in Air of a Jet Powered Aircraft

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Initial Release
March 18, 2004

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1.0 Introduction

A MATLAB m-file, VMCAv1.m, has been written to calculate the single engine minimum control speed (V_{MCA}) of a jet powered aircraft. The code calculates V_{MCA} for a given bank angle, rudder deflection, aileron deflection, asymmetric thrust and a range of user input aircraft weights. The output is a plot of minimum control speed in knots versus aircraft weight. Version 1 of this code is not interactive and requires the user to change the inputs at the head of the m-file before program execution. This report describes the theory behind the calculation of V_{MCA}, the program inputs and outputs, and some recommendations on the interpretation of the results.

2.0 Nomenclature

Symbols

b	wing span in feet
C_{Deng}	engine windmilling drag coefficient
d_i	engine inlet diameter in feet
FAR	Federal Airworthiness Regulations
l_t	asymmetric thrust moment arm in feet
M	Mach number
S_{ref}	reference wing area in square feet
T_{maxsl}	maximum asymmetric thrust in pounds
V	velocity in feet/second
V_{MCA}	single engine minimum control speed in knots of true airspeed
V_n/V	mean flow velocity in the engine nozzle exit
W	weight in pounds
_	sideslip angle in degrees (aircraft nose left positive)
_$_{ail}$	total aileron deflection in degrees (right wing down turn positive)
_$_{rud}$	rudder deflection in degrees (trailing edge left positive)
_	bank angle in degrees (right wing down positive)
_	density in slugs/ft^3

Stability Derivatives

C_l	rolling moment coefficient
C_N	yawing moment coefficient
C_Y	sideforce coefficient
$C_{l\_}$	rolling moment coefficient per degree of sideslip (/deg)
$C_{N\_}$	yawing moment coefficient per degree of sideslip (/deg)
$C_{Y\_}$	sideforce coefficient per degree of sideslip (/deg)
C_{l_ail}	rolling moment coefficient per degree of aileron deflection (/deg)
C_{N_ail}	yawing moment coefficient per degree of aileron deflection (/deg)
C_{Y_ail}	sideforce coefficient per degree of aileron deflection (/deg)
C_{l_rud}	rolling moment coefficient per degree of rudder deflection (/deg)
C_{N_rud}	yawing moment coefficient per degree of rudder deflection (/deg)
C_{Y_rud}	sideforce coefficient per degree of rudder deflection (/deg)

3.0 Minimum Control Speed Theory

The minimum control speed in air (V_{MCA}) is defined as the minimum calibrated airspeed where it is still possible to maintain control (directional, lateral and longitudinal) of an aircraft with one engine inoperative and the other at takeoff thrust. FAR 23.149 requires that minimum control speed be demonstrated with maximum takeoff thrust on one engine, the flaps in the takeoff position, the gear retracted and the aircraft's weight and center of gravity at the most unfavorable location. A maximum of 5° of bank is allowed during the maneuver. In this static V_{MCA} condition, the aircraft's sideforce, yawing moment and rolling moment are balanced if a constant heading can be maintained with one engine inoperative. Figure 1 shows the aircraft's attitude and control positions during the static V_{MCA} maneuver with a right engine failure.

The sideforce, yawing moment and rolling moment equations are given in (1), (2) & (3).

$$C_Y = C_{Y\beta}\beta + C_{Y\delta ail}\delta_{ail} + C_{Y\delta rud}\delta_{rud} + \frac{W\sin\phi}{\frac{1}{2}\rho V^2 S_{ref}} = 0 \tag{1}$$

$$C_N = C_{N\beta}\beta + C_{N\delta ail}\delta_{ail} + C_{N\delta rud}\delta_{rud} + \frac{T_{\max sl}\, l_t}{\frac{1}{2}\rho V^2 b S_{ref}} + C_{Deng}\frac{l_t}{b} = 0 \tag{2}$$

$$C_l = C_{l\beta}\beta + C_{l\delta ail}\delta_{ail} + C_{l\delta rud}\delta_{rud} = 0 \tag{3}$$

To calculate minimum control speed, a MATLAB routine was used to find the airspeed, rudder deflection and aileron deflection which balanced the sideforce, yawing moment and rolling moment equations for a given user input aircraft weight, thrust and bank angle.

For illustration, (1) & (2) can be rearranged to solve for the sideslip angle and aileron deflection required to balance the sideforce and rolling moment equations, respectively.

$$\beta = \left[-C_{Y\delta ail}\delta_{ail} - C_{Y\delta rud}\delta_{rud} - \frac{W\sin\phi}{\frac{1}{2}\rho V^2 S_{ref}} \right] \Big/ C_{Y\beta} \tag{4}$$

$$\delta_{ail} = -\left[C_{l\beta}\beta + C_{l\delta rud}\delta_{rud} \right] / C_{l\delta ail} \tag{5}$$

Equations (2), (4) & (5) indicate the following:

1. The higher the asymmetric thrust, the larger the yawing moment imbalance.
2. The higher the aircraft weight, the larger the aircraft sideslip angle. The yawing moment equation shows that more sideslip is beneficial for countering the asymmetric yawing moment due to thrust. Consequently, lighter aircraft weights will yield less sideslip angle and therefore higher V_{MCA} speeds.
3. At higher aircraft weights, sideslip angles during the maneuver can become large. This could possibly make minimum control speed limited by the amount of available aileron deflection.

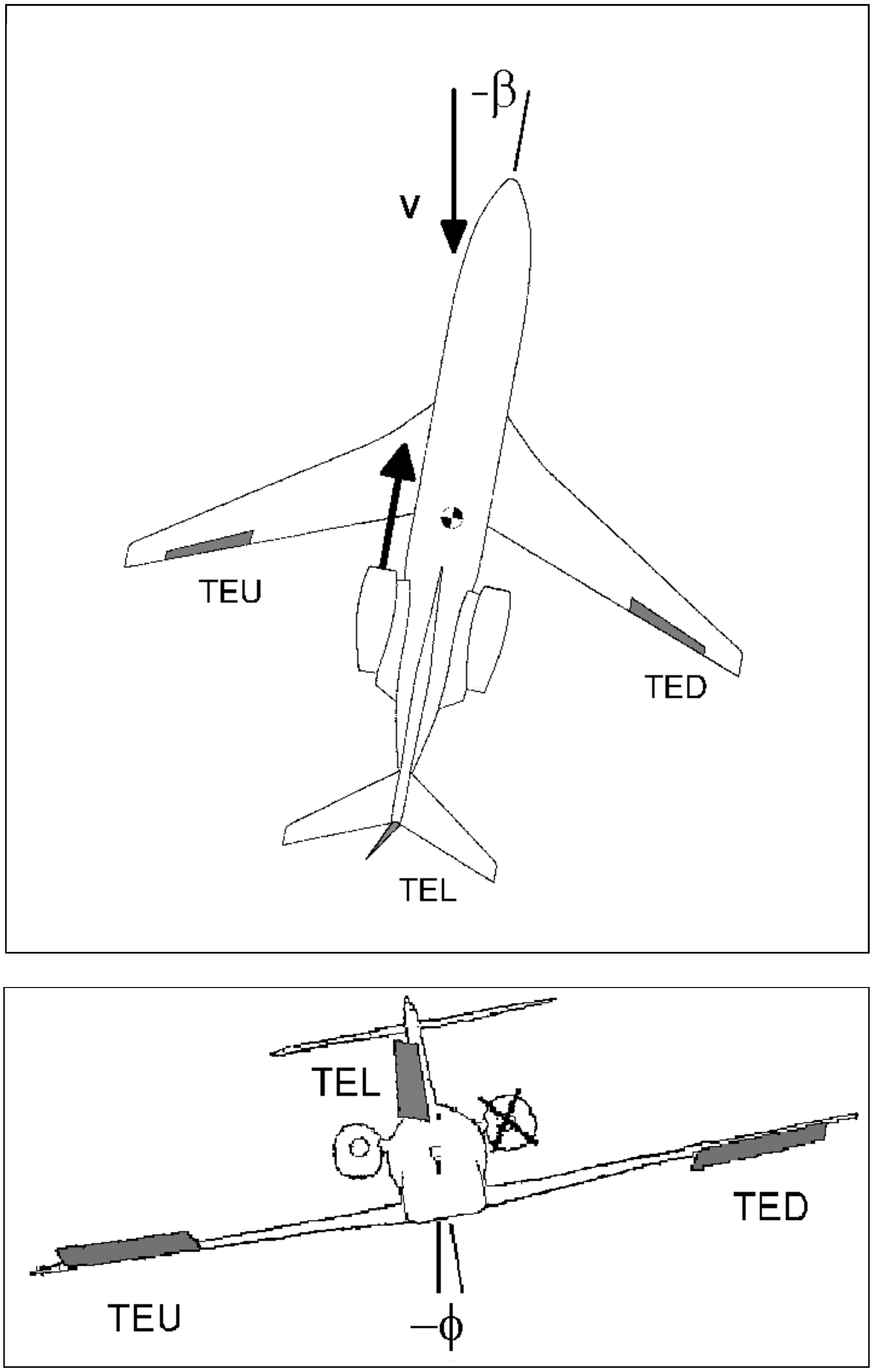

Figure 1. Aircraft Attitude and Control Positions During the Static V_{MCA} Maneuver (Right Engine Failure)

4.0 Program Overview

VMCAv1.m is a MATLAB m-file that calculates an aircraft's minimum control speed for a range of weights specified by the user. The program first computes a V_{MCA} speed based on a fixed maximum rudder deflection at each specified weight. Starting at the minimum weight, the code calculates the required aileron deflection and airspeed (V_{MCA}) that will balance the sideforce, yawing moment and rolling moment equations. As the code steps up in weight from the minimum weight input by the user, it will reach the weight where aileron deflection and rudder deflection are both at their maximum values. The code will then start at the maximum user input weight with the ailerons set to maximum deflection and calculate a V_{MCA} speed and required rudder deflection. The code will then march down in weight with aileron deflection fixed until rudder deflection reaches its maximum value. The code uses Kramer's rule to solve the system of equations, i.e. (1), (2) & (3). A plot of V_{MCA} speed versus aircraft weight is then output to the user.

4.1 Program Inputs

The input initial conditions are listed in Table 1. The code is set up to do the V_{MCA} calculation for a right engine failure. Therefore, bank angle is negative, rudder deflection is positive and aileron deflections are negative. Refer to Figure 1 for the aircraft's attitude and control positions for this set of initial conditions. The user inputs bank angle (phi), the maximum weight for the calculation (wmax), the minimum weight for the calculation (wmin) and the weight step size (dw). The code solves Equations (1), (2) & (3) using a simple matrix inversion technique (Kramer's Rule) for each weight increment. The code is very fast, so dw can be set for several hundred iterations with only a few seconds of CPU time required. The final user inputs are maximum rudder deflection (drmax), maximum aileron deflection (damax) and the maximum thrust at sea level (Tmaxsl) on the remaining engine.

Note that the code is set up to calculate minimum control speed for a right engine failure, so bank angle (phi) must be negative for a left wing down roll toward the good engine. Aileron deflection must be negative for a left wing down roll and rudder deflection must be positive for a trailing edge left deflection.

```
%------Initial Conditions for a Right Engine Failure----------------
phi = -5;                 % bank angle in degrees, FARs allow 5 deg bank toward good engine
wmax = 640000;            % maximum aircraft weight in pounds for VMCA calculation
wmin = 440000;            % minimum aircraft weight in pounds for VMCA calculation
dw = 2000;                % weight increment in pounds for VMCA calculation
drmax =  15;              % maximum rudder deflection in degrees, must be a positive number
damax =  -25;             % maximum aileron deflection in degrees, must be a negative number
Tmaxsl = 50000;           % maximum asymmetric thrust in pounds
```

Table 1. VMCAv1.m Required Initial Conditions

The required aircraft geometry inputs appear in Table 2. The moment arm to the inoperative engine (l_t), the aircraft's reference wing area (s) and wing span (b) and the engine inlet diameter (di) are required. Engine inlet diameter is used to calculate the wind-milling drag of the inoperative engine using the method outlined in Torenbeek Appendix G-8[1], see Equation (6).

$$C_{Deng} = \left[0.0785 d_i^{\,2} + \frac{2}{1+0.16M^2}\frac{\pi}{4} d_i^{\,2} \frac{V_n}{V}\left(1 - \frac{V_n}{V}\right)\right] \Big/ S_{ref} \qquad (6)$$

Note that a high bypass engine ($V_n/V = 0.92$) and a low fixed Mach number of 0.2 are assumed in the calculation of wind-milling drag in the code. Reference 2 contains more information on the calculation of engine wind-milling drag. The V_{MCA} calculation is made at sea level, as required by the FARs, so density (dsl) is the sea level value.

```
%------Aircraft Geometry--------------------------------------------
lt = 68.5 ;                 % asymmetric thrust moment arm in feet
s = 5500;                   % reference wing area in square feet
b = 195.7;                  % wing span in feet
di = 8.4;                   % engine inlet diameter in feet
dsl = .002377;              % air density at sea level in slugs per cubic foot
```

Table 2. VMCAv1.m Required Aircraft Geometry

The final inputs are the aircraft's lateral/directional stability derivatives. The required inputs are listed in Table 3. These stability derivatives can be estimated using VLM methods, the USAF Stability & Control DATCOM or the STAB.f program described in Reference 2. Note that the signs of the rudder power derivatives cndr, cldr and cydr are the opposite of those output by the STAB.f code described in Reference 2. This was done to keep with standard stability & control sign convention. The aircraft's maximum lift coefficient (clmax) is required so that a stall speed line can be put on the final V_{MCA} output plot for reference.

```
%------Stability Derivatives----------------------------------------
cnbeta =  0.002618;         % yawing moment coefficient per degree of sideslip
clbeta = -0.003857;         % rolling moment coefficient per degree of sideslip
cybeta = -0.016756;         % side force coefficient per degree of sideslip
cnda =     0.000112;        % yawing moment coefficient per degree of aileron deflection
clda =     0.000805;        % rolling moment coefficient per degree of aileron deflection
cyda =    0.0;              % side force coefficient per degree of aileron deflection
cndr =   -0.001902;         % yawing moment coefficient per degree of rudder deflection
cldr =     0.000122;        % rolling moment coefficient per degree of rudder deflection
cydr =    0.003054;         % side force coefficient per degree of rudder deflection
clmax = 1.6;                % aircraft maximum lift coefficient for this test condition
```

Table 3. VMCAv1.m Required Aircraft Stability Derivatives

4.2 Program Outputs

The primary output of VMCAv1.m is a plot of minimum control speed in knots of true airspeed versus aircraft weight. A typical output plot is given in Figure 2. Notice that the V_{MCA} solution has two distinct branches. The left branch is the rudder limited solution. Along this branch, rudder deflection is the constant user specified maximum value and aileron deflection varies as weight increases from the minimum value. The right branch is the aileron limited solution. Along this branch, aileron deflection is the constant user specified maximum value and rudder deflection increases as weight decreases from the maximum value. Rudder and aileron deflections are both at there maximum values where the two branches meet.

For reference, the code also plots stall speed versus aircraft weight. Weight (pounds), sideslip angle (degrees), rudder deflection (degrees), aileron deflection (degrees), minimum control speed (KTAS) and stall speed (KTAS) are also stored in two data files which can be viewed in the MATLAB workspace. RUDLIM contains these variables for the rudder limited solution and AILLIM contains these variables for the aileron limited solution.

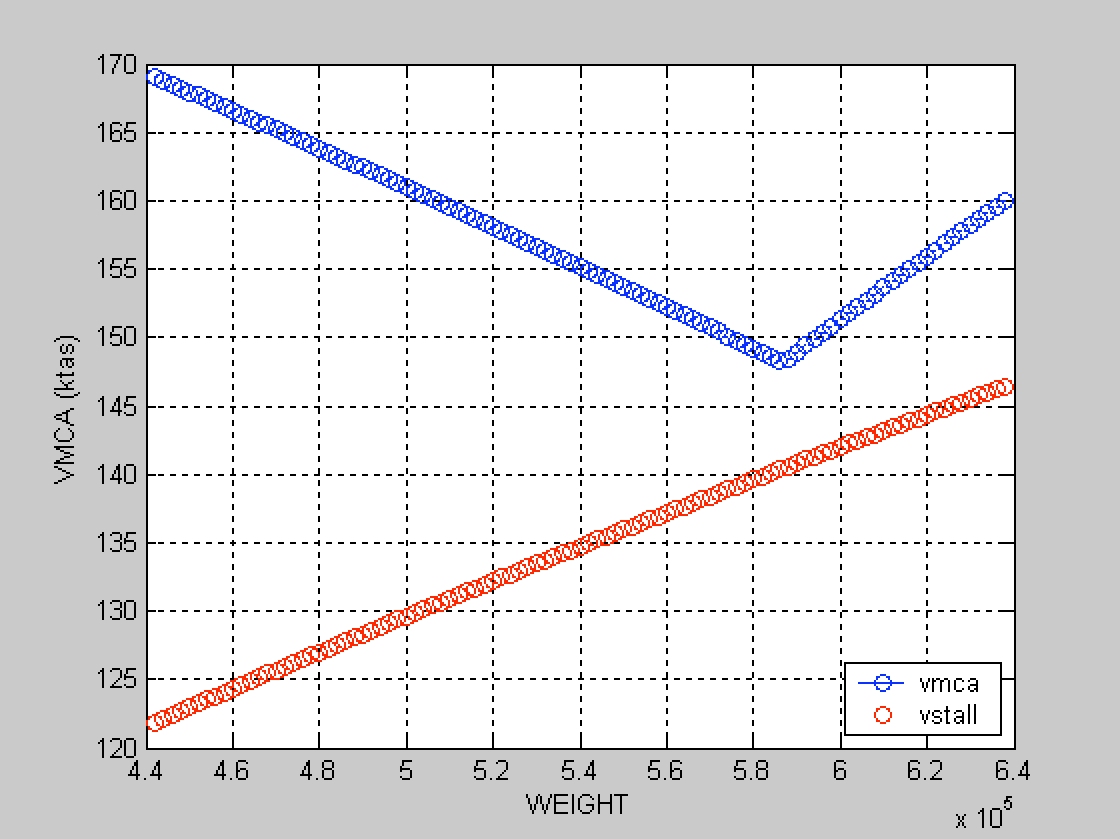

Figure 2. VMCAv1.m Output Display

5.0 Conclusions

The sample aircraft inputs used in Tables 1, 2 & 3 are for the Boeing 747-100 test case given in Reference 2. Minimum control speed was calculated for a failure of the right outboard engine. Maximum rudder and aileron control throws and the lateral/directional stability derivatives were taken directly from Reference 2. Note that the signs of the rudder power stability derivatives cndr, cldr and cydr were changed to match standard stability & control convention.

The output plot shown in Figure 2 is for the 747-100 test case. The plot shows two distinct branches to the V_{MCA} solution, a rudder limited solution and an aileron limited solution. Note that the code may not always show two distinct branches. In fact, if the test case would have been run for weights from 580,000 pounds down to 440,000 pounds, the aileron limited solution would have never appeared. In general, V_{MCA} speed is defined by the maximum rudder deflection at light weight conditions. Takeoff speed schedules (where V_{MCA} is important) are usually set as functions of percentage of stall speed. Figure 2 shows that the lightweight V_{MCA} speed is a much higher percentage of stall speed ($1.38V_{STALL}$ at 440,000 lbs.) than the heavy weight solution ($1.09\ V_{STALL}$ at 640,000 pounds).

A last word of caution, this code only considers control surface deflection (rudder and aileron) limited minimum control speeds. V_{MCA} can also be defined by control force limits. The FAR dictated 150 pound rudder pedal force limit may reduce the amount of usable rudder deflection and therefore may define V_{MCA} speed for aircraft with non-powered control systems. Accurate control surface hinge moment coefficients are required to calculate control forces during the V_{MCA} maneuver. These coefficients are generally not available during the preliminary design stage, so it is assumed that the control surface will be aerodynamically balanced (or boosted) to meet the FAR criteria.

6.0 References

1. Torenbeek, Egbert, *Synthesis of Subsonic Airplane Design,* Delft University Press, Delft, Holland, 1982.

2. Grasmeyer, Joel, "Stability and Control Derivative Estimation and Engine-Out Analysis," Virginia Tech Department of Aerospace and Ocean Engineering Report, VPI-AOE-254, January 1998.

3. Cavanaugh, Michael A., "Design Study to Reduce the Single Engine Minimum Control Speed of the SJ30-2 Twin-Engine Business Jet," SAE 1999-01-1601, April 1999.

Appendix A. VMCAv1.m Program Listing

```
%-----------------------------------------------------------------------------------------
%
%    VMCAv1.m  Program to Calculate the Minimum Control Speed in Air
%
%              Mike Cavanaugh  3/08/04
%
%-----------------------------------------------------------------------------------------
clear

%------Initial Conditions for a Right Engine Failure-----------------
phi = -5;                  % bank angle in degrees, FARs allow 5 deg bank toward good engine
wmax = 640000;             % maximum aircraft weight in pounds for VMCA calculation
wmin = 440000;             % minimum aircraft weight in pounds for VMCA calculation
dw = 2000;                 % weight increment in pounds for VMCA calculation
drmax =  15;               % maximum rudder deflection in degrees, must be a positive number
damax =  -25;              % maximum aileron deflection in degrees, must be a negative number
Tmaxsl = 50000;            % maximum asymmetric thrust in pounds

%------Aircraft Geometry--------------------------------------------
lt = 68.5 ;                % asymmetric thrust moment arm in feet
s = 5500;                  % reference wing area in square feet
b = 195.7;                 % wing span in feet
di = 8.4;                  % engine inlet diameter in feet
dsl = .002377;             % air density at sea level in slugs per cubic foot

%------Stability Derivatives----------------------------------------
cnbeta =  0.002618;        % yawing moment coefficient per degree of sideslip
clbeta = -0.003857;        % rolling moment coefficient per degree of sideslip

cybeta = -0.016756;        % side force coefficient per degree of sideslip
cnda =     0.000112;       % yawing moment coefficient per degree of aileron deflection
clda =     0.000805;       % rolling moment coefficient per degree of aileron deflection
cyda =    0.0;             % side force coefficient per degree of aileron deflection
cndr =   -0.001902;        % yawing moment coefficient per degree of rudder deflection
cldr =     0.000122;       % rolling moment coefficient per degree of rudder deflection
cydr =    0.003054;        % side force coefficient per degree of rudder deflection
clmax = 1.6;               % aircraft maximum lift coefficient for this test condition

%-----Calculate drag of windmilling engine (Ref. Torenbeek G-8)------
cdeng = (.1934*di^2)/s;

%-----Loop in Weight for Rudder Limited VMCA------------------------
i = 1;
for W = wmin:dw:wmax

 %-----Find Rudder Limited Solution----------------------------------------
  dr = drmax;
  A11 = cybeta;
  A12 = cyda;
  A13 = 0;
  A14 = ((2*W)/(dsl*s))*sin(phi/57.3);
  A21 = cnbeta;
  A22 = cnda;
  A23 = cdeng*lt/b;
  A24 = (2*Tmaxsl*lt)/(s*b*dsl);
  A31 = clbeta;
```

```
  A32 = clda;
  A33 = 0;
  A34 = 0;
  A41 = 0;
  A42 = 0;
  A43 = 1;
  A44 = 0;

  B11 = -cydr*dr;
  B21 = -cndr*dr;
  B31 = -cldr*dr;
  B41 = 1;

  A = [A11 A12 A13 A14; A21 A22 A23 A24; A31 A32 A33 A34; A41 A42 A43 A44];
  B = [B11; B21; B31; B41];

  c = inv(A)*B;

  beta = c(1,1);
  da = c(2,1);
  vfts = c(4,1)^-.5;
  q = .5 * dsl * vfts^2;
  vkts = vfts/1.15*(3600/5280);
  vskts = 0.5928854*((2*W)/(dsl*s*clmax))^.5;

  rudlim(i,1) = W;
  rudlim(i,2) = beta;
  rudlim(i,3) = dr;
  rudlim(i,4) = da;
  rudlim(i,5) = vkts;
  rudlim(i,6) = vskts;

  if abs(da) >= abs(damax)
     break
  end

  %-----plot results-------------------------------------------------
  if i>1
   hold on
   plot(W,vkts,'-ob',W,vskts,'or')
   grid on
   box on
   xlabel('WEIGHT')
   ylabel('VMCA (ktas)')
   legend ('vmca','vstall', 4)
  end
  i=i+1;
end

%----Loop in Weight for Aileron Limited Solution-
i = 1;
for W = wmax:-dw:wmin

  %-----Find Aileron Limited Solution--------------------------------
  da = damax;
  AA11 = cybeta;
```

```
  AA12 = cydr;
  AA13 = 0;
  AA14 = ((2*W)/(dsl*s))*sin(phi/57.3);
  AA21 = cnbeta;
  AA22 = cndr;
  AA23 = cdeng*lt/b;
  AA24 = (2*Tmaxsl*lt)/(s*b*dsl);
  AA31 = clbeta;
  AA32 = cldr;
  AA33 = 0;
  AA34 = 0;
  AA41 = 0;
  AA42 = 0;
  AA43 = 1;
  AA44 = 0;

  BB11 = -cyda*da;
  BB21 = -cnda*da;
  BB31 = -clda*da;
  BB41 = 1;

  AA = [AA11 AA12 AA13 AA14; AA21 AA22 AA23 AA24; AA31 AA32 AA33 AA34; AA41 AA42 AA43
AA44];
  BB = [BB11; BB21; BB31; BB41];

  cc = inv(AA)*BB;

  beta = cc(1,1);
  dr = cc(2,1);
  vfts = cc(4,1)^-.5;
  q = .5*dsl*vfts^2;
  vkts = vfts/1.15*(3600/5280);
  vskts = 0.5928854*((2*W)/(dsl*s*clmax))^.5;

  aillim(i,1) = W;
  aillim(i,2) = beta;
  aillim(i,3) = dr;
  aillim(i,4) = da;
  aillim(i,5) = vkts;
  aillim(i,6) = vskts;

  if abs(dr) >= abs(drmax)
    break
  end

  %-----plot results-------------------------------------------------
  if i>1
   hold on
   plot(W,vkts,'-ob',W,vskts,'-or')
  end
  i=i+1;
end
```