

**MULTIDISCIPLINARY OPTIMIZATION OF AN HSCT WING
USING A RESPONSE SURFACE METHODOLOGY**

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1. Introduction

Aerospace vehicle design is traditionally divided into three phases: conceptual, preliminary, and detailed. Each of these design phases entails a particular level of accuracy and computational expense. While there are several computer programs which perform inexpensive conceptual-level aircraft multidisciplinary design optimization (MDO), aircraft MDO remains prohibitively expensive using preliminary- and detailed-level analysis tools. This occurs due to the expense of computational analyses and because gradient-based optimization requires the analysis of hundreds or thousands of aircraft configurations to estimate design sensitivity information.

A further hindrance to aircraft MDO is the problem of *numerical noise* which occurs frequently in engineering computations. Computer models produce numerical noise as a result of the incomplete convergence of iterative processes, round-off errors, and modeling errors. Such numerical noise is typically manifested as a high frequency, low amplitude variation in the results obtained from the computer models¹. Optimization attempted using noisy computer models may result in the erroneous calculation of design sensitivities and may slow or prevent convergence to an optimal design.

Statistical techniques including design of experiments (DOE)² and response surface (RS) methodologies³ may be used to overcome the computational costs and numerical noise problems inherent in aircraft MDO. DOE methods provide statistically sound strategies for selecting series or batches of computational experiments, which allow the researcher to separate the true measured *responses* in a set of data from sources of experimental error. Further, DOE strategies establish a framework for the selection of a limited number of expensive computational experiments and provide guidelines for the accuracy and type of information which can be gained from them. Response surface modeling was developed to analyze the results of the batches of experiments and to produce models (curve fits in one dimension or surface fits in multiple dimensions) which relate the dependence of the observed responses to the values of the independent variables. There is a growing interest in the application of DOE and RS methods to aerospace design as discussed in Reference 4.

To demonstrate how DOE and RS methodologies can aid in aircraft MDO the design optimization of a High-Speed Civil Transport (HSCT) has been considered. In this application a *full factorial experimental design* was used to define an initial batch of HSCT configurations which were analyzed using the most inexpensive conceptual-level analysis methods. After screening out any grossly unsuitable HSCT configurations a *D-optimal experimental design* was used to identify a select few of the HSCT configurations for which the more expensive, preliminary-level analyses were conducted. RS models were then created from this data for several aerodynamic drag components which were known to be sources of numerical noise. The RS models were then used in the optimization process in lieu of the computationally expensive and noisy analysis methods. Thus, the computational costs of aircraft MDO were transferred from the optimization stage to the DOE stage of aircraft design. In this strategy, parallel computing may be applied to

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efficiently perform the numerous aircraft configuration evaluations specified by the DOE methods. The parallel computing methods used in this study are detailed in Reference 5.

Our research group at Virginia Tech has developed a method to overcome the difficulties of aerospace vehicle MDO that we term *customized response surface modeling*. The customized RS modeling technique employs DOE and RS methods in the aircraft design process and was developed by Giunta⁶ et al. (1995) to investigate the feasibility of using response surface models in a simple HSCT wing shape optimization problem. The present work extends the work of Reference 6 by employing some of the techniques used by Kaufman⁷ et al. developed in related HSCT design optimization efforts. The main emphasis of this paper is to demonstrate the utility of DOE methods in aircraft design and to show that polynomial RS models can filter out numerical noise. Using the RS models, wing optimizations for the HSCT will be performed and the results compared to traditional optimizations conducted without the RS models. Through these comparisons it will be shown that the use of DOE and RS modeling reduces some of the problems associated with aircraft system MDO.

2. HSCT Design Optimization

2.1 Design Tools

A suite of conceptual-level and preliminary-level tools has been created for HSCT analysis and design. These tools include several public domain software packages obtained from NASA along with numerous in-house developed software modules. A description of these analysis and optimization tools is given in Reference 8. For the example HSCT wing design problem to be presented here, Carlson's supersonic panel code⁹, the Harris wave drag code¹⁰, and several in-house codes were used to estimate cruise aerodynamics. In addition, the Flight Optimization System (FLOPS)¹¹ was used to obtain aircraft weights and several in-house codes were used to evaluate aircraft performance at both cruise and landing conditions. Currently, detailed-level analysis tools (e.g., Euler/Navier-Stokes aerodynamic analyses¹²) are not included due to their high computational expense. However, the customized RS modeling method is easily adaptable to aircraft design involving conceptual- and preliminary-level analyses alone or to aircraft design incorporating all three levels of analysis tools.

2.2 HSCT Wing Design Example Problem

To develop and validate the customized RS modeling method a simple five-variable HSCT wing design problem was investigated. This five-variable problem is a subset of the "full" HSCT design problem developed by researchers at Virginia Tech which employs twenty-nine design variables to describe an HSCT wing/fuselage/tail/engine configuration⁸. The objective of the five-variable HSCT wing design problem was to minimize the Takeoff Gross Weight (TOGW) of the HSCT configuration within the allowable limits of the design variables. These variables were root chord (C_{root}), tip chord (C_{tip}), inboard leading-edge sweep angle (Λ_{LE}), thickness-to-chord (t/c) ratio, and fuel weight (W_{fuel}) which had initial values of $C_{root} = 185.0 \text{ ft}$, $C_{tip} = 10.0 \text{ ft}$, $\Lambda_{LE} = 75.0^\circ$, $t/c \text{ ratio} = 2.0\%$, and $W_{fuel} = 315,000 \text{ lb}$. The wing planform and airfoil section definitions are shown in Figure 1. To uniquely define the wing planform from the four wing variables, the length of the leading-edge from the wing/fuselage junction to the leading-edge break was held constant as was the outboard leading-edge sweep angle. For the airfoil section definitions, the leading-edge radius was held constant along with the chordwise location of maximum thickness.

In the five-variable example problem the fuselage definition was held fixed with a fuselage length of 300 ft and an internal volume of 23,720 ft³. Additionally, the vertical tail area was held fixed at 700 ft², and the engine thrust was held constant at 39,000 lb per engine. A horizontal tail was not considered for this aircraft. The mission profile also was simplified from the "full" HSCT aircraft design problem to include only a supersonic cruise leg and some landing considerations related to aircraft weight. The altitude for the Mach 2.4 supersonic cruise mission was held constant at 65,000 ft. Landing constraints on both the overall lift coefficient (C_L) and eighteen wing section lift coefficients (C_l) were examined for emergency landing situations. The forty-two nonlinear constraints for this problem consist of both geometric constraints (e.g., all wing chords $\geq 7.0 \text{ ft}$), and aerodynamic/performance constraints (e.g., C_L at landing ≤ 1 , and range $\geq 5,500 \text{ n.mi.}$). The five-variable HSCT wing design optimization problem may be expressed as

$$\min_{x \in R^5} TOGW(x), \text{ subject to } g_i(x) \leq 0, \quad i = 1, \dots, 42, \quad (1)$$

where x is the five-dimensional vector of design variables, and g is the forty-two-dimensional vector of nonlinear inequality constraints.

3. Statistical Methods

A response surface methodology is a formal process combining elements of experimental design, regression analysis, and analysis of variance used to aid in process and/or project optimization³. In many RS modeling applications, either linear or quadratic polynomials are assumed to accurately model the selected response. Although this is certainly not true for all cases, RS modeling becomes prohibitively expensive when cubic and higher-order polynomials are chosen for experiments involving several variables.

A quadratic response surface model has the form

$$y = c_o + \sum_{1 \leq j \leq m} c_j x_j + \sum_{1 \leq j < k \leq m} c_{jk} x_j x_k, \quad (2)$$

where y is the observed response, x_j are the m design variables, and c_o , c_j , and c_{jk} are the unknown polynomial coefficients. Note that there are $n = (m + 1)(m + 2)/2$ coefficients in this quadratic polynomial. To estimate the unknown polynomial coefficients in the RS model, at least $p \geq n$ observed response values must be available. Under such conditions, the estimation problem may be formulated in matrix notation as $Y \approx \mathbf{X}c$, where Y is the p by 1 vector of observed response values, \mathbf{X} is a p by n matrix of constants assumed to have rank n , and c is the n by 1 vector of unknown coefficients to be estimated. The least squares solution to $Y \approx \mathbf{X}c$ is $\hat{c} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T Y$. Typically values for p of at least $1.5n$ to $2.5n$ are required to produce response surface models which accurately approximate the underlying function.

4. Numerical Noise Issues

In past research related to our current MDO efforts, convergence difficulties were encountered in the aerodynamic-structural optimization of the HSCT⁸. The convergence problems were traced to *numerical noise* in the computation of aerodynamic lift and drag components. This oscillatory behavior created numerous local optima in the design space which resulted in either convergence failure or slow convergence to a locally optimal design when using gradient-based optimization methods.

The lift and drag values predicted by some of the aerodynamic analysis tools are sensitive to slight changes in the aircraft design. This sensitivity is illustrated in Figure 2 which shows supersonic volumetric wave drag calculated for an HSCT at Mach 2.4. Here, the analysis method was the Harris wave drag code¹⁰ where all of the HSCT design variables were fixed, with the exception of the wing semi-span which was varied from fifty to one hundred feet. As the semi-span increased, numerical noise was created by a high frequency variation in the calculated wave drag values. Physically, wave drag should change smoothly as the wing shape is varied. Note that this numerical noise occurs on an extremely small scale with variations in wave drag on the order of 0.02 to 0.1 drag counts (drag in counts = $C_D \times 10^{-4}$). The Harris wave drag code has an accuracy of approximately 0.5 to 1.0 drag counts and was not developed with optimization in mind. Hence, wave drag variations of 0.02 to 0.1 counts were considered inconsequential by the original software developers.

In addition to the noisy supersonic volumetric wave drag ($C_{D_{wave}}$) analyses, two components of the supersonic drag-due-to-lift analyses ($C_{D_{lift}}$) are affected by numerical noise as well. Here, drag-due-to-lift is calculated as $C_{D_{lift}} = (1/C_{L\alpha} - k_t C_T/C_L^2) C_L^2$, where $C_{L\alpha}$ is the supersonic lift curve slope, C_T/C_L^2 is the leading-edge thrust term, and k_t is an attainable leading-edge thrust factor. The numerical noise in the drag-due-to-lift evaluation may be attributed to numerical noise in the $C_{L\alpha}$ and C_T/C_L^2 calculations as is discussed in Reference 6. As with wave drag, the numerical noise in $C_{D_{lift}}$ is within the intended accuracy of the analysis method. For this five-variable HSCT wing design study the customized RS modeling method will be employed to model three parameters: $C_{D_{wave}}$, $C_{L\alpha}$, and C_T/C_L^2 .

5. HSCT Wing Design Using Customized RS Modeling

The construction of the response surface models may be viewed as a series of steps to be completed before the aircraft system optimization is performed. This methodology is detailed in Reference 5.

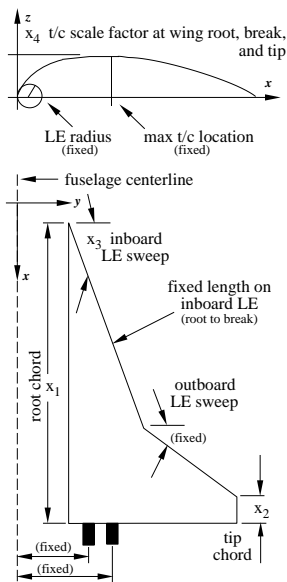


Figure 1. HSCT design variables.

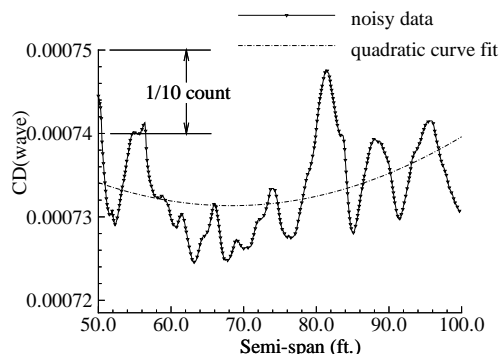


Figure 2. Numerical noise in wave drag.

5.1 Conceptual-Level HSCT Analyses

A 5^5 full factorial design based on DOE methods was constructed around an initial HSCT configuration. The minimum and maximum values of each of the five design variables were selected so as not to exceed the capabilities of the analysis tools (e.g., forward swept wings cannot be analyzed). The range of each of the five design variables was (148.0 ft, 222.0 ft), (8.0 ft, 12.0 ft), (68.25°, 81.75°), (1.6%, 2.4%), and (305,550 lb, 324,450 lb) for $C_{D_{root}}$, C_{tip} , Λ_{LE} , t/c ratio, and W_{fuel} , respectively.

The 3,125 (5^5) full factorial HSCT configurations were analyzed using the inexpensive conceptual-level analysis tools. These 3,125 HSCT configurations were then screened to eliminate from consideration any grossly infeasible designs, i.e., those which exceeded any of the geometric constraints by more than five percent or any of the aerodynamic constraints by ten percent. After screening, 1,860 HSCT configurations remained and the data from these 1,860 analyses were used to construct *conceptual-level* response surface models for $C_{D_{wave}}$, $C_{L\alpha}$, and C_T/C_L^2 . Adjusted R^2 values, which measure how accurately a response surface model fits the given data, were calculated as 0.9910, 0.9992, and 0.9956 for $C_{D_{wave}}$, $C_{L\alpha}$, and C_T/C_L^2 , respectively. A value of adjusted $R^2 = 1.0$ denotes an exact fit of the RS models to the data.

5.2 Preliminary-Level HSCT Analyses

In the next step of the customized RS modeling process, the commercial statistical package JMP¹³ was used to create a D -optimal experimental design¹⁴ comprised of fifty HSCT configurations out of the 1,860 remaining HSCT candidates. These fifty designs were then evaluated using the preliminary-level analysis tools and *preliminary-level* response surface models were constructed for $C_{D_{wave}}$, $C_{L\alpha}$, and C_T/C_L^2 using the resulting analysis data. Adjusted R^2 values for the preliminary-level response surface models were calculated as 0.9953, 0.9965, and 0.9908 for $C_{D_{wave}}$, $C_{L\alpha}$, and C_T/C_L^2 , respectively.

5.3 Optimization Using Response Surface Models

The preliminary-level RS models could then be used in the HSCT system optimization to replace the noisy drag calculations. To prevent the optimizer from exploiting potential weaknesses in the response surface models, ten side constraints were applied (upper and lower bounds on each variable) to keep the optimizer within the design space defined for this study.

Figures 3 and 4 show optimization results for the Case 1 HSCT wing designs obtained with and without using the customized RS modeling method. As shown in Figure 3 for the Case 1 optimal wing obtained using customized RS models, there were slight changes in the planform geometry between the initial and optimal designs. Specifically, the initial and final designs were (185.0 ft, 10.0 ft, 75.0°, 2.0%, 315,000 lb) and (165.2 ft, 8.6 ft, 72.9°, 2.2%, 305,550 lb), respectively. These design changes result in a savings of approximately 7,000 lb in the structural weight of the wing and about 10,000 lb of fuel weight. These weight savings

more than offset the slight decrease in aerodynamic efficiency in the optimal wing which had a maximum lift-to-drag ratio, $(L/D)_{max}$, of 9.76 compared to $(L/D)_{max}$ of 9.85 for the initial wing.

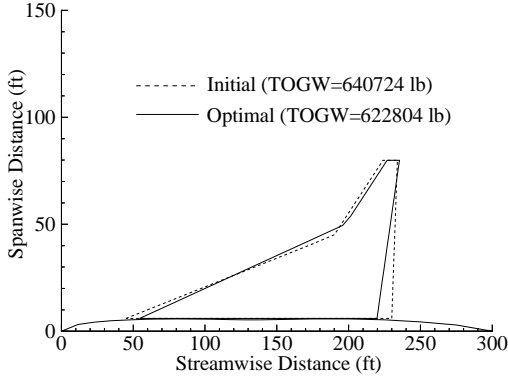


Figure 3. Case 1 initial and optimal designs with RS models.

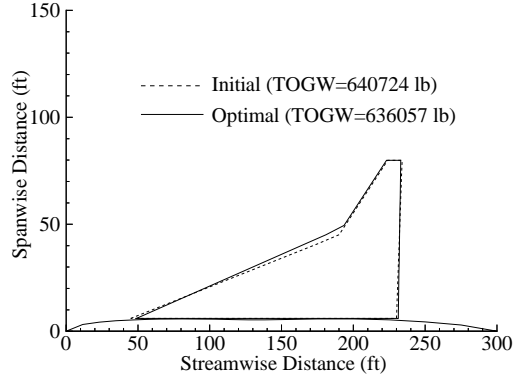


Figure 4. Case 1 initial and optimal designs without RS models.

In contrast, Figure 4 shows almost no change between the initial and optimal wing designs when the RS models were not used. This occurred because numerical noise in the wave drag and drag-due-to-lift calculations led to numerical noise in the TOGW calculations. This numerical noise in TOGW created local minima in the design space which prevented the optimizer from locating the globally optimal wing design. The consequence of not using the RS models was a 14,000 *lb* difference in TOGW between the globally optimal design and the locally optimal design.

The effects of numerical noise are further demonstrated in the Case 2 optimization results (Figures 5, 6) and the Case 3 optimization results (Figures 7, 8). In both Case 2 and Case 3, the use of RS modeling allowed the optimizer to identify the globally optimal wing design. However, if the RS models were not employed, the optimizer became trapped in one of the locally optimal designs.

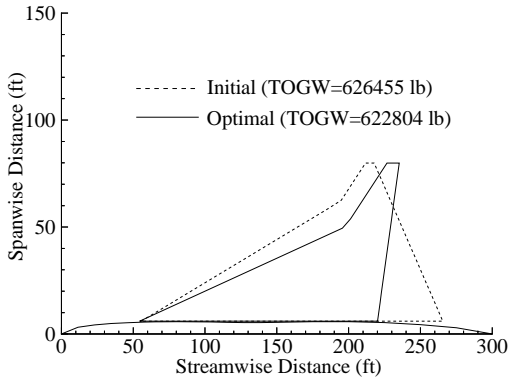


Figure 5. Case 2 initial and optimal designs with RS models.

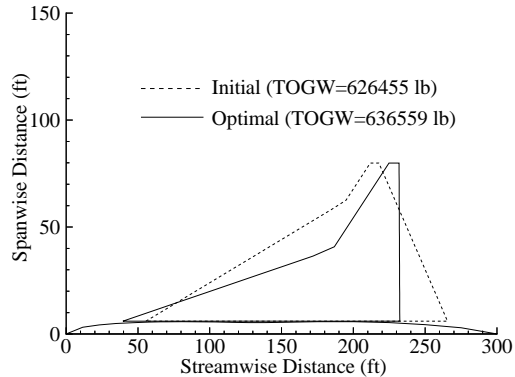


Figure 6. Case 2 initial and optimal designs without RS models.

6. Conclusions

The HSCT optimization results show that the customized response surface modeling method is successful in applying DOE and RS modeling methods to aircraft design. Specifically, by employing the customized RS modeling method, a globally optimal HSCT wing design was identified whereas only locally optimal HSCT wing designs were found when optimization was attempted without using the customized RS models.

Current efforts focus on the extension of the customized RS modeling method to a more realistic HSCT design optimization problem involving ten variables and a mission profile involving both subsonic and supersonic segments.

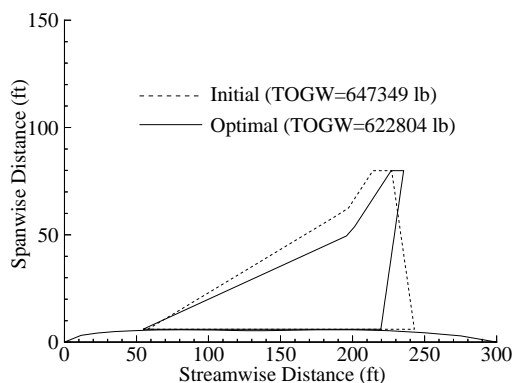


Figure 7. Case 3 initial and optimal designs with RS models.

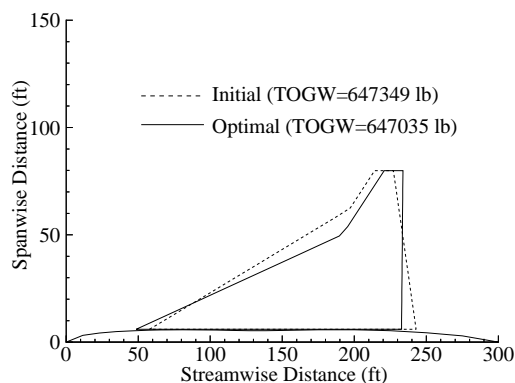


Figure 8. Case 3 initial and optimal designs without RS models.

Acknowledgments

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