E.7 STDATM

This subroutine computes the 1976 standard atmosphere. It is used in program FRICTION. It covers an altitude range from sea level to 86 kilometers (282,152 ft.). The results are found in either English or metric units depending on the value of one of the input flags. The 1976 and 1962 standard atmospheres are identical for the first 51 kilometers above sea level.

Method of Computation

Given the geometric altitude Z_{in} (in dimensions of either meters or feet), convert to kilometers. The geopotential altitude H is then found from:

$$H = \frac{Z}{1 + \frac{Z}{r_0}}$$

where r_0 = 6356.766 kilometers (the radius of the Earth in kilometers) and $Z = C_1 Z_{in}$, where C_1 = 0.001 if Z_{in} is in meters, and C_1 = 0.0003048 if Z_{in} is in feet. The 1962 standard atmosphere used a much more complicated and slightly more accurate relationship.

The inverse relation is given by

$$Z = \frac{H}{1 - \frac{H}{r_0}} .$$

Once the geopotential altitude is found, the temperature is computed. The standard day temperature profile is defined by seven layers, where within each layer the temperature is found by the linear relation (T is given in degrees Kelvin):

$$T = T_{b_i} + L_{m_i}\left(H - H_{b_i}\right)$$

and T_{b_i}, L_{m_i} and H_{b_i} are the values at the base of the particular layer. The following table defines these constants, as well as the ratio of pressure to sea level pressure, which is also needed.

i	H_{b_i} (Km)	T_{b_i} (°K)	L_{m_i} (°K/Km)	*P/Psl*	*Z*(ft.)
1	0.	288.15	-6.5	1.0	0
2	11.	216.65	0.0	2.2336×10^{-1}	36,152.
3	20.	216.65	+1.0	5.4032×10^{-2}	65,824
4	32.	228.65	+2.8	8.5666×10^{-3}	105,518
5	47.	270.65	0.0	1.0945×10^{-3}	155,348
6	51.	270.65	-2.8	6.6063×10^{-4}	168,676
7	71.	214.65	-2.0	3.9046×10^{-5}	235,571
-	82.	-	-	-	282,152

Once the temperature is determined, the pressure is computed using the hydrostatics equation and the perfect gas law. The resulting formulas are:

$$\frac{P}{P_{sl}} = \frac{P_b}{P_{sl}}\left(\frac{T_b}{T}\right)^{\frac{K}{L_m}} \qquad L_m \neq 0$$

$$\frac{P}{P_{sl}} = \frac{P_b}{P_{sl}}\, e^{\frac{-K(H-H_b)}{T_b}} \qquad L_m = 0$$

where $K = \frac{g_o M_0}{R^*} = 34.163195$ in consistent units. The remaining fundamental property is the density, which is found using the equation of state as:

$$\frac{\rho}{\rho_{sl}} = \frac{P/P_{sl}}{T/T_{sl}}\,.$$

Additional parameters of interest in aerodynamics are:

i) The speed of sound

$$a = a_{sl}\sqrt{\frac{T}{T_{sl}}}$$

ii) The coefficient of viscosity, found from Sutherland’s Law:

$$\mu = \frac{\beta \cdot T^{3/2}}{T + S}$$

where S = 110.4°K and β depends on the system of units and is defined below.

iii) The Reynolds number per unit length and Mach:

$$\frac{R_e}{M \cdot L} = \frac{\rho a}{\mu}$$

iv) The actual temperature, pressure and density:

$$T = T_{sl}\left(\frac{T}{T_{sl}}\right)$$

$$P = P_{sl}\left(\frac{P}{P_{sl}}\right)$$

$$\rho = \rho_{sl}\left(\frac{\rho}{\rho_{sl}}\right)$$

and *v*) the dynamic pressure normalized by the Mach number:

$$\frac{q}{M^2} = \frac{\gamma}{2}P = .7P$$
.

The sea level properties and other required constants are defined in the following table.

	Metric	English
T_{sl}	288.15 °K	518.67° R
P_{sl}	1.01325×10^5 N/m^2	2116.22 lb/ft^2
ρ_{sl}	1.2250 Kg/m^3	0.0023769 slugs/ft^3
a_{sl}	340.294 m/sec	1116.45 ft/sec
μ_{sl}	1.7894×10^{-5} Kg/m/sec	0.37373×10^{-6} slugs/ft/sec
β	1.458×10^{-6} Kg/m/sec/K $^{1/2}$	3.0450963×10^{-8} slugs/ft/sec/K$^{1/2}$

The ratio of specific heats, γ, is defined to be 1.40.

User instructions: the comments in the subroutine define the input and output argument list. If the maximum altitude is exceeded, the program returns a non zero value of the validity flag.

```
      subroutine stdatm(z,t,p,r,a,mu,ts,rr,pp,rm,qm,kd,kk)
c
c    *********** 1976 STANDARD ATMOSPHERE SUBROUTINE **********
c
c      Mason's BASIC program, converted to FORTRAN - Sept. 1, 1989
c
c      kd -    = 0 - metric units
c             <> 0 - English units
c
c      kk -      0 - good return
c                1 - error: altitude out of table,
c                           do not use output
c
c      z  - input altitude, in feet or meters (depending on kd)
c
c      output:
c                                  units:     metric        English
c      t  - temp.                             deg K          deg R
c      p  - pressure                          N/m^2          lb/ft^2
c      r  - density                           Kg/m^3         slugs/ft^3
c      a  - speed of sound                    m/sec          ft/sec
c      mu - viscosity                         Kg/m/sec       slug/ft/sec
c
c      ts - t/t at sea level
c      rr - rho/rho at sea level
c      pp - p/p at sea level
c
c      rm - Reynolds number per               Re/M/m         Re/M/ft
c            Mach per unit of length
c      qm - dynamic pressure/Mach^2          N/m^2           lb/ft^2
```

```
c
      real k, h, mu, ml
      KK = 0
      K  = 34.163195
      C1 = 3.048E-04
      IF (KD .eq. 0) go to 1240
      TL = 518.67
      PL = 2116.22
      RL = .0023769
      AL = 1116.45
      ML = 3.7373E-07
      BT = 3.0450963E-08
      GO TO 1260

 1240 TL = 288.15
      PL = 101325
      RL = 1.225
      C1 = .001
      AL = 340.294
      ML = 1.7894E-05
      BT = 1.458E-06

 1260 H = C1 * Z / (1 + C1 * Z / 6356.766)
      IF (H .gt. 11.0) go to 1290
      T = 288.15 - 6.5 * H
      PP = (288.15 / T) ** ( - K / 6.5)
      GO TO 1420

 1290 IF (H .gt. 20.0) go to 1310
      T = 216.65
      PP = .22336 *  EXP ( - K * (H - 11) / 216.65)
      GO TO 1420

1310  IF (H .gt. 32.0) go to 1330
      T = 216.65 + (H - 20)
      PP = .054032 * (216.65 / T) ** K
      GO TO 1420

1330  IF (H .gt. 47.0) go to 1350
      T = 228.65 + 2.8 * (H - 32)
      PP = .0085666 * (228.65 / T) ** (K / 2.8)
      GO TO 1420

1350  IF( H .gt. 51.0) go to 1370
      T = 270.65
      PP = .0010945 *  EXP ( - K * (H - 47) / 270.65)
      GO TO 1420

1370  IF (H .gt. 71.) go to 1390
      T = 270.65 - 2.8 * (H - 51)
      PP = .00066063 * (270.65 / T) ** ( - K / 2.8)
      GO TO 1420

1390  IF (H .gt. 84.852) THEN
                              kk = 1
                              write(6,200) H
                              return
                         END IF
```

```
      T = 214.65 - 2 * (H - 71)
      PP = 3.9046E-05 * (214.65 / T) ** ( - K / 2)

1420  RR = PP / (T / 288.15)
      MU = BT * T**1.5 / (T + 110.4)
      TS = T / 288.15
      A  = AL *  SQRT (TS)
      T  = TL * TS
      R  = RL * RR
      P  = PL * PP
      RM = R * A / MU
      QM = .7 * P

  200 format('   Out of Table in StdAtm- too high !'//
     1        4x,'H =',f12.3,'  > 84.852 km'/)

      return
      end
```

The following sample program and output can be used to validate your subroutine:

```
c     main program to check stdatm

c     loop is done twice to get output
c     suitable to include in text(80 col)

c     w.h. mason, Feb. 27, 1994

      real mu

      kd      = 1

      write(6,90)

      do 10 i = 1,21
      z       = 5000.*(i-1)

      call stdatm(z,t,p,r,a,mu,ts,rr,pp,rm,qm,kd,kk)

      if (kk .ne. 0) then
                     write(6,120)
                     stop
                     endif

      write(6,100) z,t,p,r,a,mu
   10 continue

      write(6,110)
      do 20 i = 1,21
      z       = 5000.*(i-1)

      call stdatm(z,t,p,r,a,mu,ts,rr,pp,rm,qm,kd,kk)

      if (kk .ne. 0) then
                     write(6,160)
                     stop
                     endif
```

```
      write(6,120) z,ts,rr,pp,rm,qm
   20 continue

   90 format(/3x,'1976 Standard Atmosphere'//
     1        3x,'    alt      T       P        Rho',
     2        2x,'     a         Mu',
     4       /3x,'   (ft)   (deg R)  (psf)    (s/ft^3)',
     5        2x,' (f/s)  (slugs/ft/sec)')
  100 format(3x,f9.1,f8.2,f8.2,e12.4,f8.2,e12.4)
  110 format(/3x,'1976 Standard Atmosphere'//
     1        3x,'    alt    T/Tsl  R/Rsl',
     2        2x, 'P/Psl  Re/M/ft    q/M^2',
     4       /3x,'   (ft)',34x,'(lb/ft^2)')
  120 format(3x,f9.1,3f7.4,e10.3,f10.4)
  160 format(/4x,'error in return code from stdatm - pgm stops'/)
      stop
      end
```

Sample output:

```
  1976 Standard Atmosphere

      alt      T       P        Rho        a        Mu
     (ft)   (deg R)  (psf)    (s/ft^3)   (f/s)  (slugs/ft/sec)
        0.0  518.67 2116.22  0.2377E-02 1116.45  0.3737E-06
     5000.0  500.84 1760.88  0.2048E-02 1097.10  0.3637E-06
    10000.0  483.03 1455.60  0.1756E-02 1077.40  0.3534E-06
    15000.0  465.22 1194.79  0.1496E-02 1057.36  0.3430E-06
    20000.0  447.42  973.28  0.1267E-02 1036.93  0.3324E-06
    25000.0  429.62  786.34  0.1066E-02 1016.10  0.3217E-06
    30000.0  411.84  629.67  0.8907E-03  994.85  0.3107E-06
    35000.0  394.06  499.35  0.7382E-03  973.14  0.2995E-06
    40000.0  389.97  393.13  0.5873E-03  968.08  0.2969E-06
    45000.0  389.97  309.45  0.4623E-03  968.08  0.2969E-06
    50000.0  389.97  243.61  0.3639E-03  968.08  0.2969E-06
    55000.0  389.97  191.80  0.2865E-03  968.08  0.2969E-06
    60000.0  389.97  151.03  0.2256E-03  968.08  0.2969E-06
    65000.0  389.97  118.93  0.1777E-03  968.08  0.2969E-06
    70000.0  392.25   93.73  0.1392E-03  970.90  0.2984E-06
    75000.0  394.97   73.99  0.1091E-03  974.26  0.3001E-06
    80000.0  397.69   58.51  0.8571E-04  977.62  0.3018E-06
    85000.0  400.42   46.35  0.6743E-04  980.95  0.3035E-06
    90000.0  403.14   36.78  0.5315E-04  984.28  0.3052E-06
    95000.0  405.85   29.23  0.4196E-04  987.59  0.3070E-06
   100000.0  408.57   23.27  0.3318E-04  990.90  0.3087E-06

  1976 Standard Atmosphere

      alt    T/Tsl  R/Rsl  P/Psl  Re/M/ft    q/M^2
     (ft)                                 (lb/ft^2)
        0.0 1.0000 1.0000 1.0000 0.710E+07 1481.3538
     5000.0 0.9656 0.8617 0.8321 0.618E+07 1232.6129
    10000.0 0.9313 0.7386 0.6878 0.535E+07 1018.9235
    15000.0 0.8969 0.6295 0.5646 0.461E+07  836.3538
    20000.0 0.8626 0.5332 0.4599 0.395E+07  681.2936
    25000.0 0.8283 0.4486 0.3716 0.337E+07  550.4373
    30000.0 0.7940 0.3747 0.2975 0.285E+07  440.7683
```

```
    35000.0 0.7598 0.3106 0.2360 0.240E+07  349.5441
    40000.0 0.7519 0.2471 0.1858 0.191E+07  275.1887
    45000.0 0.7519 0.1945 0.1462 0.151E+07  216.6139
    50000.0 0.7519 0.1531 0.1151 0.119E+07  170.5264
    55000.0 0.7519 0.1205 0.0906 0.934E+06  134.2600
    60000.0 0.7519 0.0949 0.0714 0.736E+06  105.7186
    65000.0 0.7519 0.0747 0.0562 0.579E+06   83.2541
    70000.0 0.7563 0.0586 0.0443 0.453E+06   65.6079
    75000.0 0.7615 0.0459 0.0350 0.354E+06   51.7925
    80000.0 0.7668 0.0361 0.0276 0.278E+06   40.9574
    85000.0 0.7720 0.0284 0.0219 0.218E+06   32.4446
    90000.0 0.7772 0.0224 0.0174 0.171E+06   25.7445
    95000.0 0.7825 0.0177 0.0138 0.135E+06   20.4621
   100000.0 0.7877 0.0140 0.0110 0.107E+06   16.2903

STOP
```