W. H. Mason, June 27, 2015

Curiosity Number 5. The variation of neutral point location on wings (aero center)

Students often assume that the aerodynamic center location for a swept wing is located at the quarter chord of the mean aerodynamic chord. We know this is close for airfoils and it can be found to be exactly at the quarter chord in thin airfoil theory. Although this is very roughly true for wings, it's not exactly true. The good news is that Shevell included a chart of the aero center location variation with sweep and taper in the 2nd edition of *Fundamentals of Flight* (Fig. 13.11 on page 228). The figure didn't include forward sweep, and it showed that when the sweep of the quarter chord is zero the *ac* was located exactly at the quarter chord. Let's look at the aerodynamic center location for wings a little more carefully.

In this analysis we will examine wings with various sweep angles, and rectangular unswept wings to see the aerodynamic center variation. Lets start with Shevell's figure (credited to Ilan Kroo). That figure is reproduced here as Fig. 1. For sweep angles of 30 – 40 degrees, the aero center is aft of the quarter chord. For a taper of 0.5 (I'm not sure why he uses σ instead of λ, but it just illustrates to students that they have to accept various notations. Admittedly this is hard when first learning a subject, but a common problem.) For the λ = 0.5 case the aerodynamic center is slightly forward of the quarter chord at low sweep angles. Moreover, the curve is drawn to go to 0.25 for all the cases at zero sweep. We will see next that this is not exactly right.

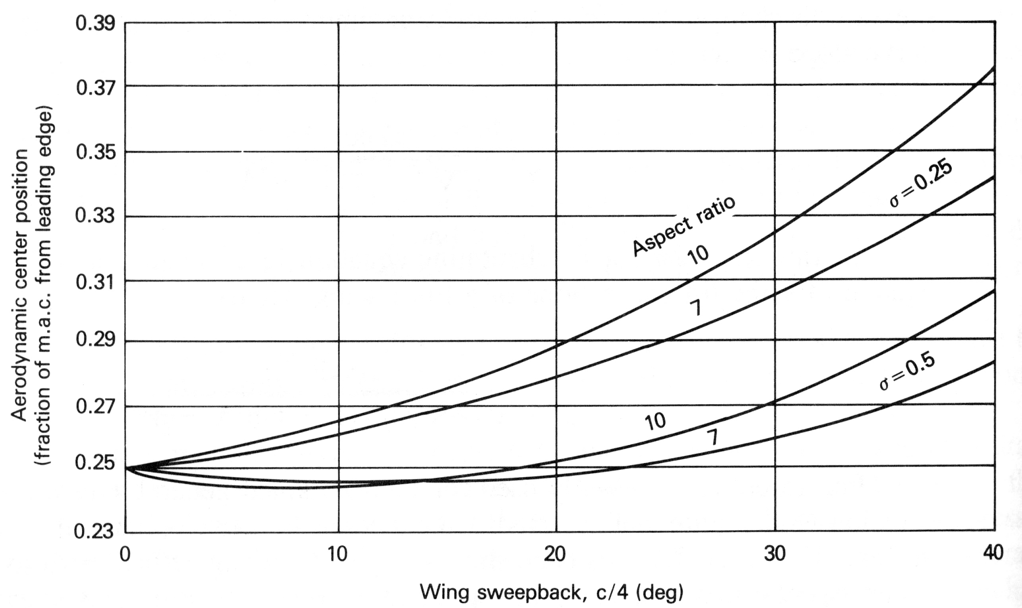

Fig. 1. The figure presented in Shevell showing the variation with sweep for aspect ratios of 7 and 10 and taper ratios of 0.25 and 0.5.

Figure 2 presents the results of my calculation of the aerodynamic center location for a couple of similar cases. Notably, since I had some involvement with forward swept wings I included forward sweep. Naturally the results were curious. I already knew that the *ac* was shifted between forward and aft sweep. The results are from a “toy” vlm method I coded up (and validated by comparison with the results Lan presented for the Warren 12 wing).

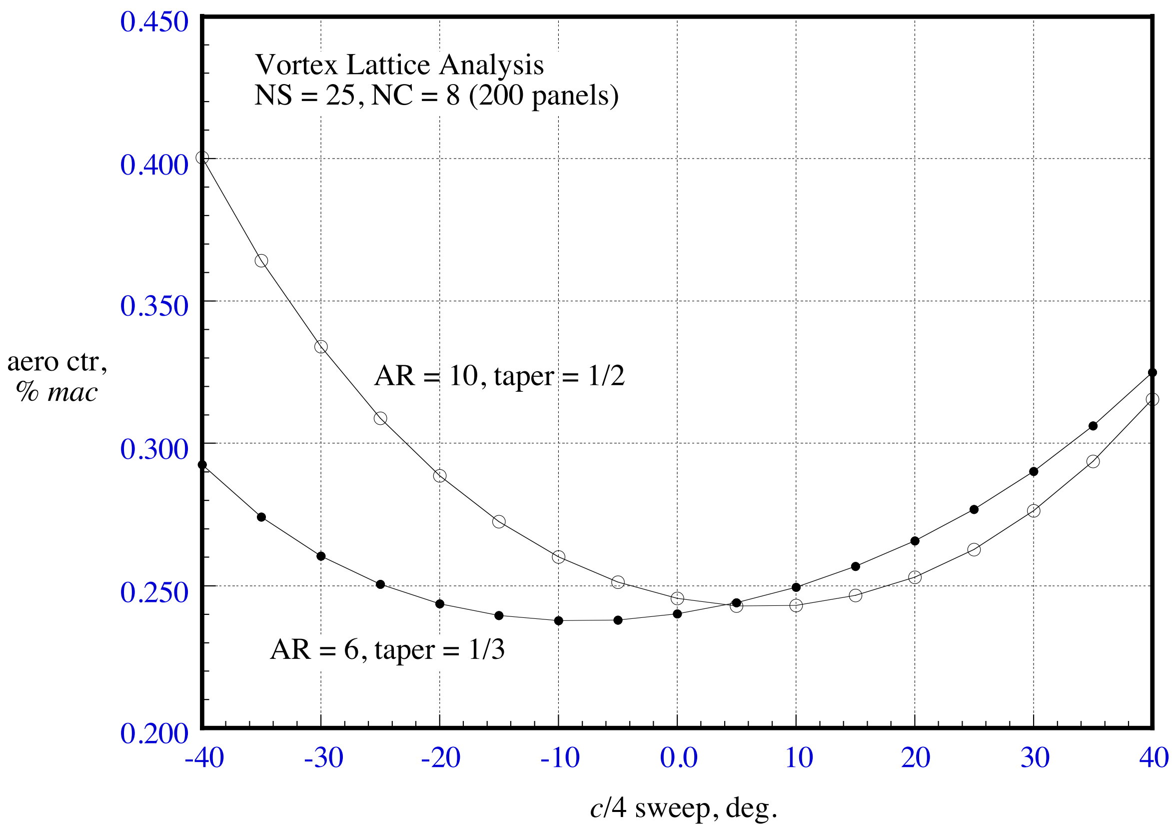

Figure 2. Aerodynamic center variation for wings with forward and aft sweep.

First note that the curves clearly don’t go to 0.25 at zero sweep. For AR = 10 and λ = 0.5 it appears we reproduce the result contained in Fig. 1, except that the curve doesn’t go exactly through 0.25 at zero sweep. The *ac* is the furthest forward at around 7 or 8 degrees sweep. Also, it’s not symmetric between forward and aft sweep. For forward sweep the *ac* is all the way back at 40% *mac* at -40 deg. sweep. Next, note that the curve for AR = 6 and λ = 0.333 demonstrates a different trend. The forward location of the *ac* occurs at about -10 degrees, and the *ac* is not as far aft for the forward swept case at -40 degrees as it is for the +40 deg. sweep case.

We conclude that the *ac* location for forward swept wings is much more dependent on the specific planform than for aft swept wings. At present I don’t have a physical explanation.

Because the aero center doesn’t seem to be exactly at 0.25 for a finite wing we look at this in a little more detail. Before examining the aero center variation for a rectangular unswept wing as the aspect ratio varies, we note the results given by Bill Rodden in a *JA* note a few years ago, “Methods for Calculating the Subsonic Aerodynamic Center of Finite Wings” (2003). He considered an AR = 20 rectangular wing and tabulated the sensitivity of the results to various numbers of panels. For 10 spanwise and 5 chordwise panels he got the *ac* at 24.77% of the chord, and my toy code got 24.79%. I would say pretty close, and when I upped the number of panels to 20 spanwise and 10 chordwise the answer changed slightly to 24.74%. So I have confidence in my calculation.

Although I calculated the *ac* variation for various aspect ratios for an unswept rectangular wing, my calculation simply confirmed the results given in the figure from Schlichting and Truckenbrodt, *Aerodynamics of the Airplane*, given here as Fig. 3.

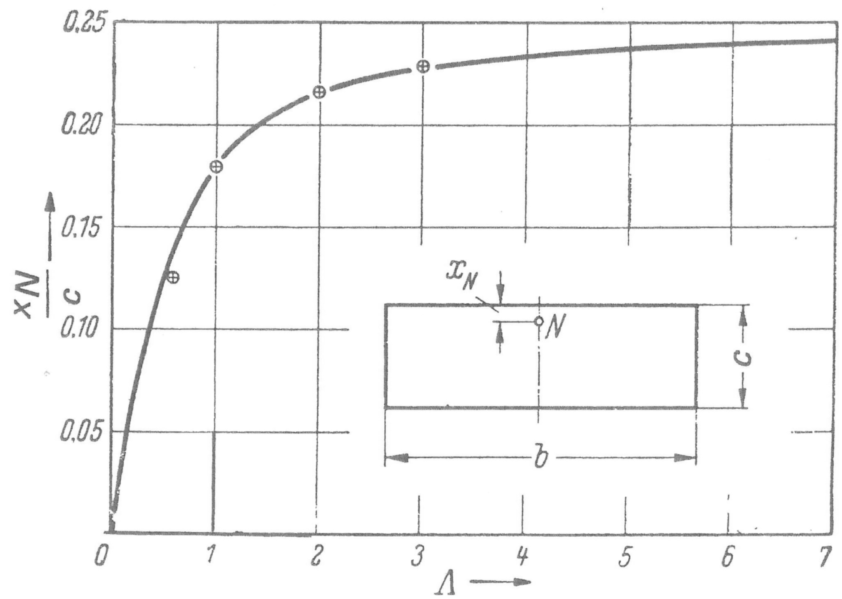

Figure 3. Variation of the aerodynamic center location (they refer to this as the neutral point) as the aspect ratio varies (Schlichting and Truckenbrodt). They use *Λ* as the symbol for *AR*, I have no idea why.

This figure demonstrates that the aero center approaches the quarter chord *very* slowly as the aspect ration increases (Rodden's *AR* = 20 case only got to 0.2477). Perhaps of more interest to me was the forward movement of the *ac* as the aspect ratio decreases. I had discovered this trend years ago when making pretest estimates for an "Inboard Wing" concept that we tested in the wind tunnel. I found this figure when hunting for something to validate the results I got using VLMpc. Curiously for my toy vlm code the numerical solution breaks down for *AR* = 0.6 and below. Above that *AR* it agrees with the results in Fig, 3.

The Takeaway:

We conclude from these results that the aerodynamic center changes more than the classic ¼ chord rule of thumb suggests, and we need to remember these results when doing aerodynamics.