Curiosity Number 22. $x\ln(x)$ as x goes to zero

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Looking at thin airfoil theory and numerous other analytic aerodynamic theories, especially involving small perturbation analysis, some form of the term $x\ln(x)$ seems to come up frequently. It is well known that as x goes to zero this term goes to zero, even though $\ln(x)$ becomes singular. In fact this is often described as a "weak" singularity. I don't recall seeing plots of this function, so I thought I'd make a few. In particular, I thought I'd see if there was an approximation near zero. So far I haven't found one. Anyway, these are some plots I made around zero. We take a look at both the term and the gradient. Yep, the term goes to zero and the slope goes to negative infinity. I made the plots at two different scales.

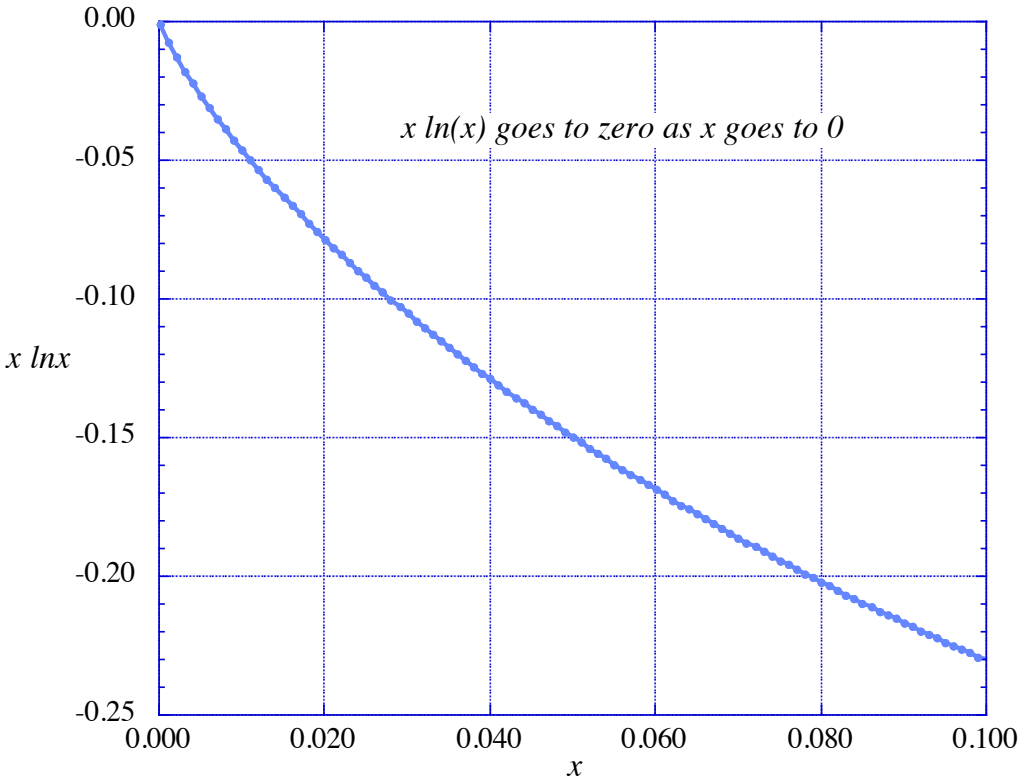

Figure C22-1a. $x\ln(x)$ from $x = 0$ to $x = 0.10$.

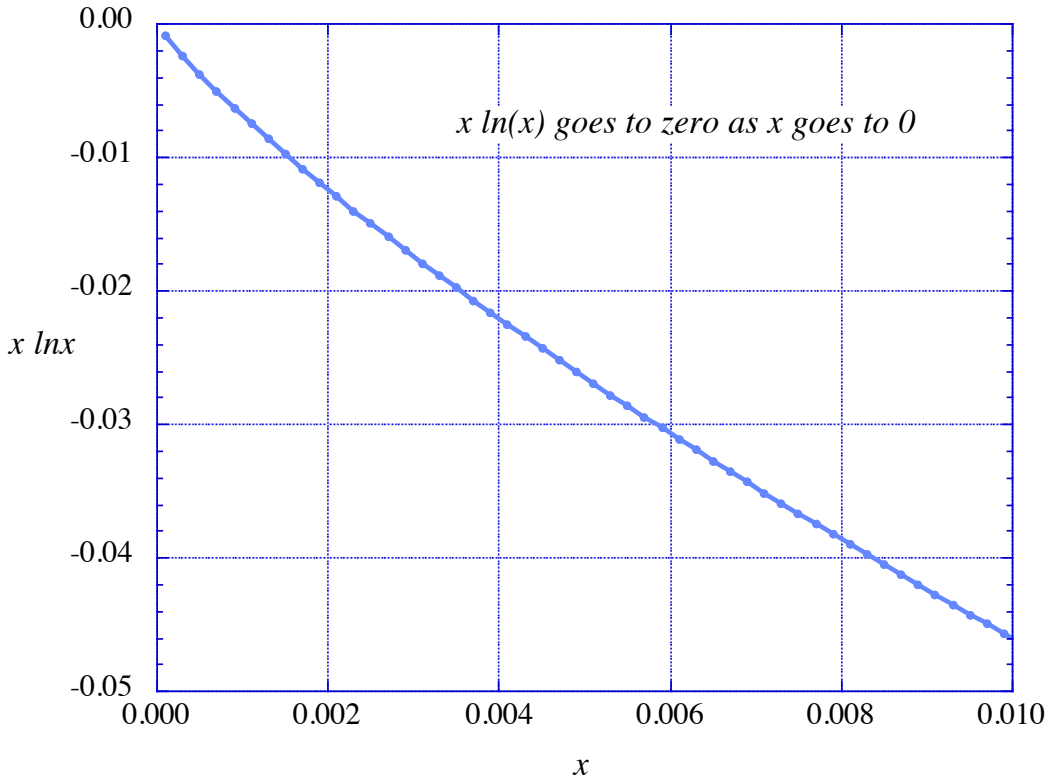

Figure C22-1b. $x\ln(x)$ with an expanded scale $x = 0$ to x = 0.01.

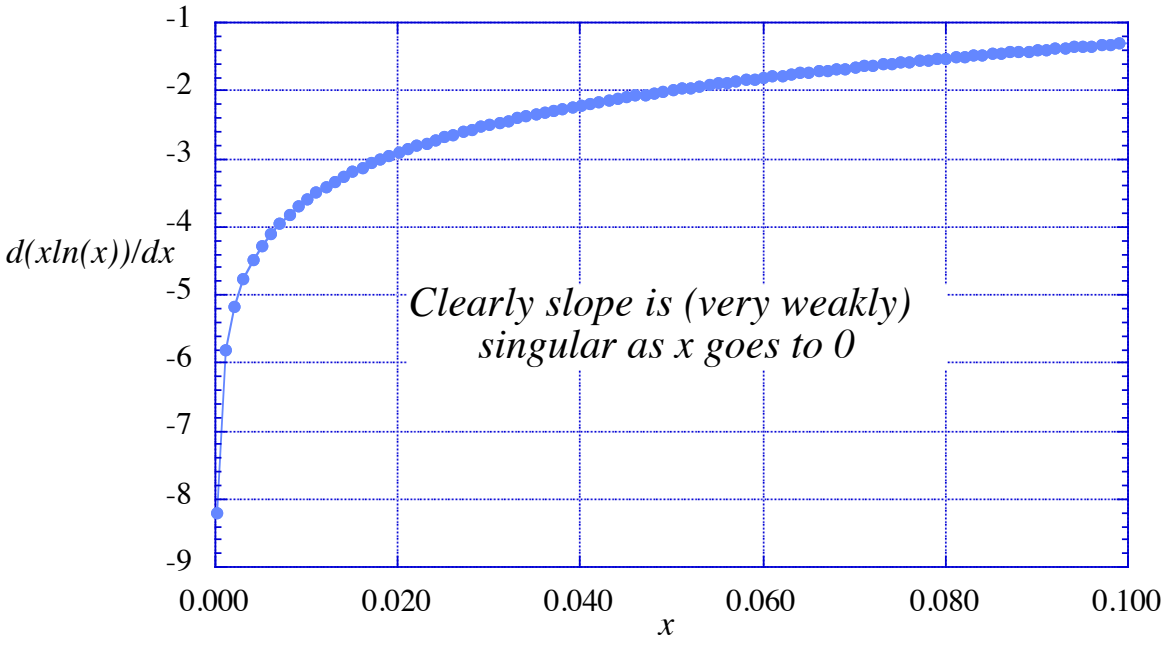

Figure C22-2a. The derivative of $x\ln(x)$ from $x = 0$ to $x = 0.10$.

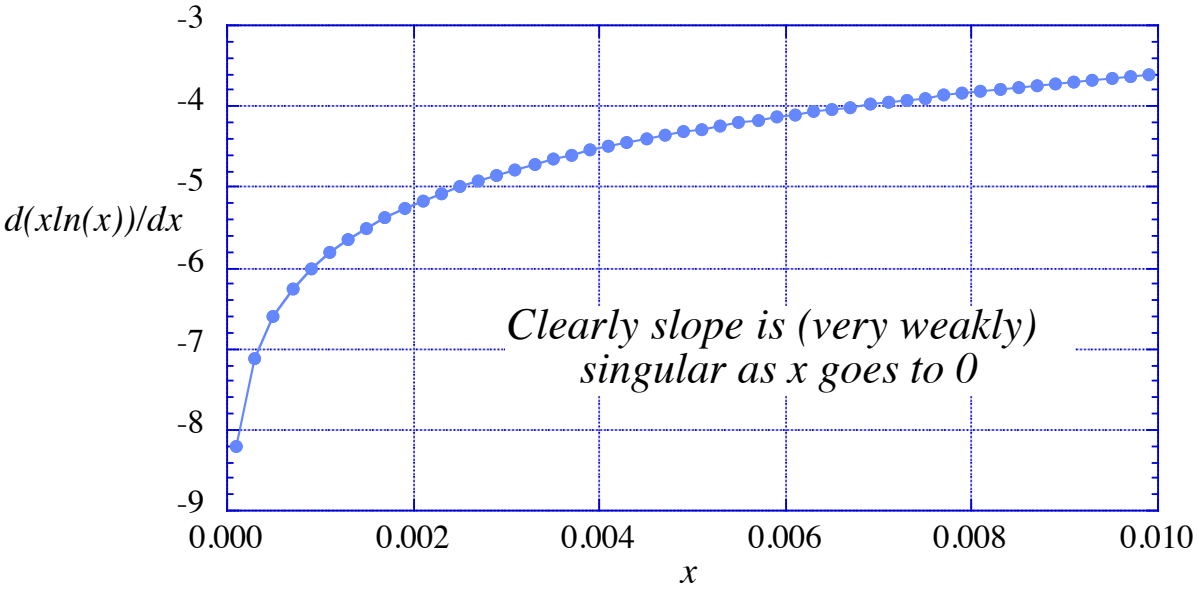

Figure C22-2b. The derivative of $x\ln(x)$ with an expanded scale $x = 0$ to x = 0.01.

This is particularly curious since van Dyke's *Perturbation Methods in Fluid Mechanics* has lots of cases where this form occurs, but no plots. When he wrote the book today's software wasn't available. It's a shame that small perturbation and asymptotic analysis aren't employed in aerodynamics anymore. It's a good way to get important insight into what controls aerodynamic flowfields.