Curiosity Number 18. Some Skin Friction Drag Estimate Oddities

We've seen plenty of plots for skin friction coefficient variations both for Reynolds Number and Mach number. When doing some of these for myself I found that plotting outside the usual range of values (letting the plotting package do its own thing) leads to surprises (mostly obvious). So I'm sharing these. When I refer to Bertin and Cummings it's the 5th Ed, 2009.

Low speed skin friction drag trends with Reynolds number.

First, let's look at the classic curve for skin friction drag with Reynolds number. Figure C18-1 shows the usual case. This was done to compare with the drag of an NACA 0012 airfoil. Thus the drag coefficient is twice the total skin friction drag coefficient (sometimes called the average skin friction coefficient). This is the chart I show my Configuration Aero class.
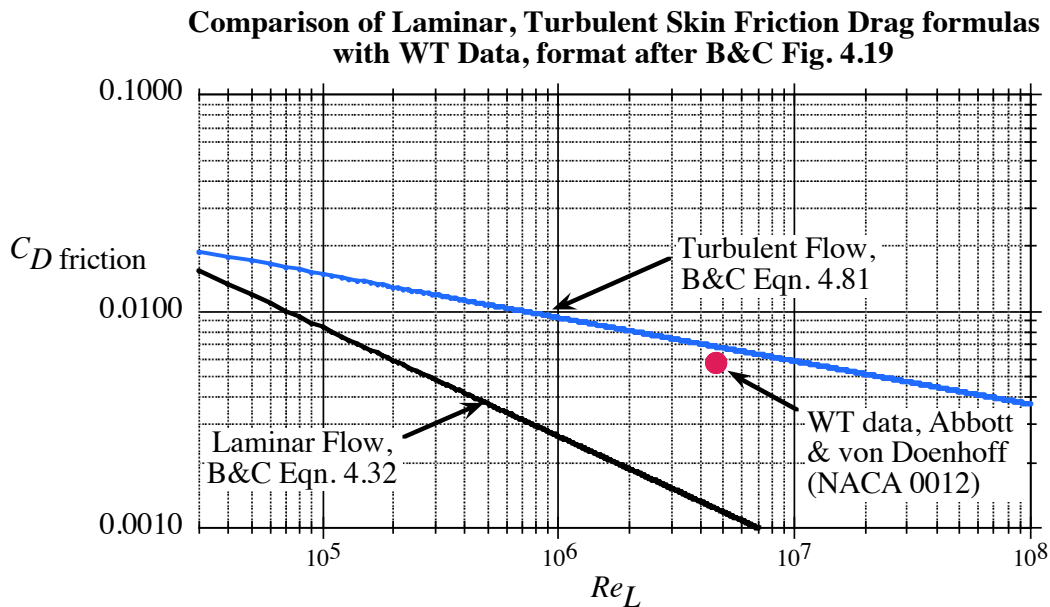

Figure C18-1. Classic laminar and turbulent skin friction drag coefficient curves

Next, take a look at what happens if we extend the range of Reynolds numbers. The first time I made the plot this was what the plotting package wanted to do. Figure C18-2 shows what happens. The curves cross. I had never thought about this although it's obvious. Of course this is way outside of the range of validity for the turbulent formula. I thought this was interesting. Why did it take me so long to discover this? I'm surprised a student didn't find this, this is the kind of discovery they're really good at (and they also assume that it's real).

**Comparison of Laminar, Turbulent Skin Friction Drag formulas
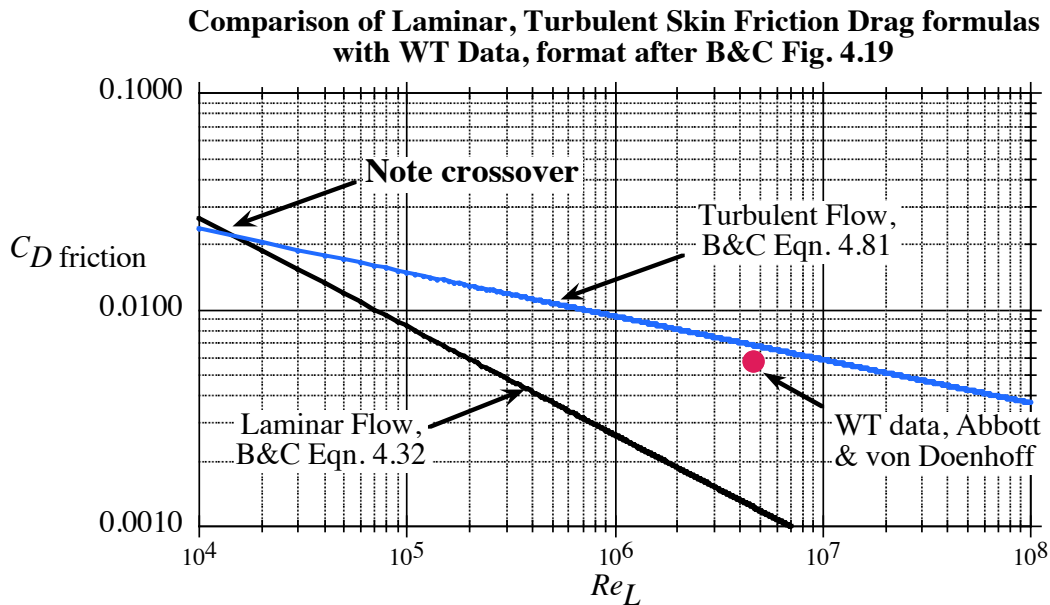with WT Data, format after B&C Fig. 4.19**

Figure C18-2. Skin friction drag coefficient trends including really low Reynolds numbers.

Using the Prandtl formula, B&C Eqn. 4.81, the intersection can be found to be at a Reynolds number of 15,117, way below any reasonable value for turbulent flow. This points to the need to include bounds on the range of validity when providing these formulas.

Because Russ lists the accuracy of the Prandtl formula as +/- 25% on page 178 (quoting White?) I decided to compare this formula with the others he lists. In part I did this after looking at other books where the Prandtl formula didn't look that bad to me. Also, most comparisons are for local skin friction, not the total skin friction. And frankly I usually use the Prandtl formula to make quick estimates.

I plotted the comparisons using different scales for the drag coefficient. I thought that might provide more insight into the differences. I would say it didn't. Nevertheless, I'm including both. The figure using the log scale is Figure C18-3, and the figure using the "normal" scale is Figure C18-4. They both show similar results, with all the formulas being in the best agreement in the range from 1 to 10 million, as would be expected since that's the range where there would be the most experimental data. I'd say yes, for a submarine don't use Prandtl's formula, it's the one that is departing from the others at very high Reynolds numbers. The formulas are so close I had to use a legend, although it's still hard to tell which is which.
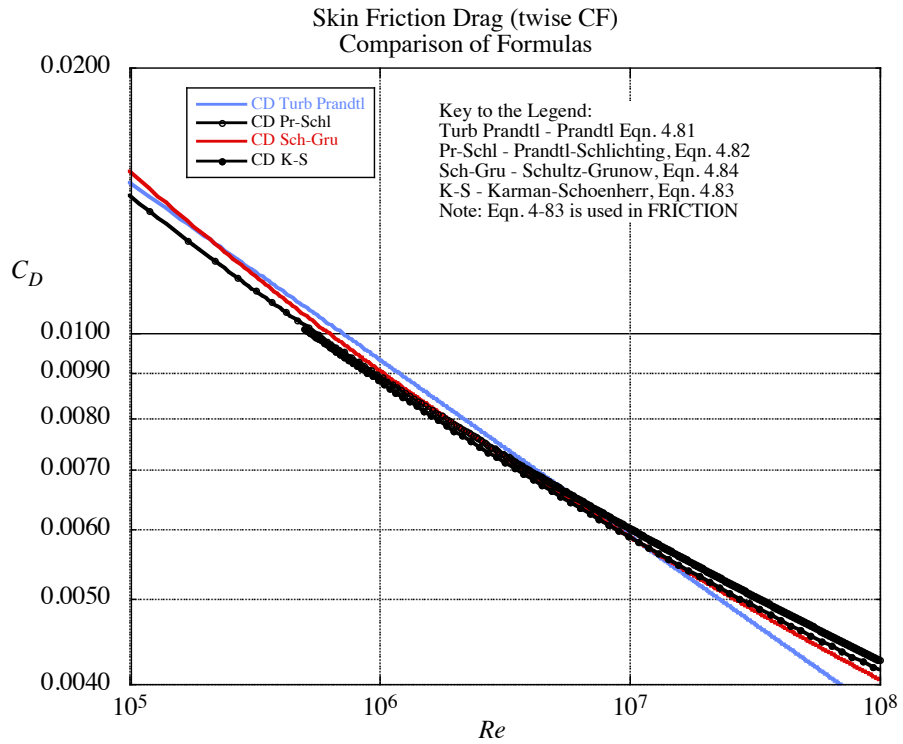
Figure C18-3. Skin friction drag coefficient formula comparisons with a log scale
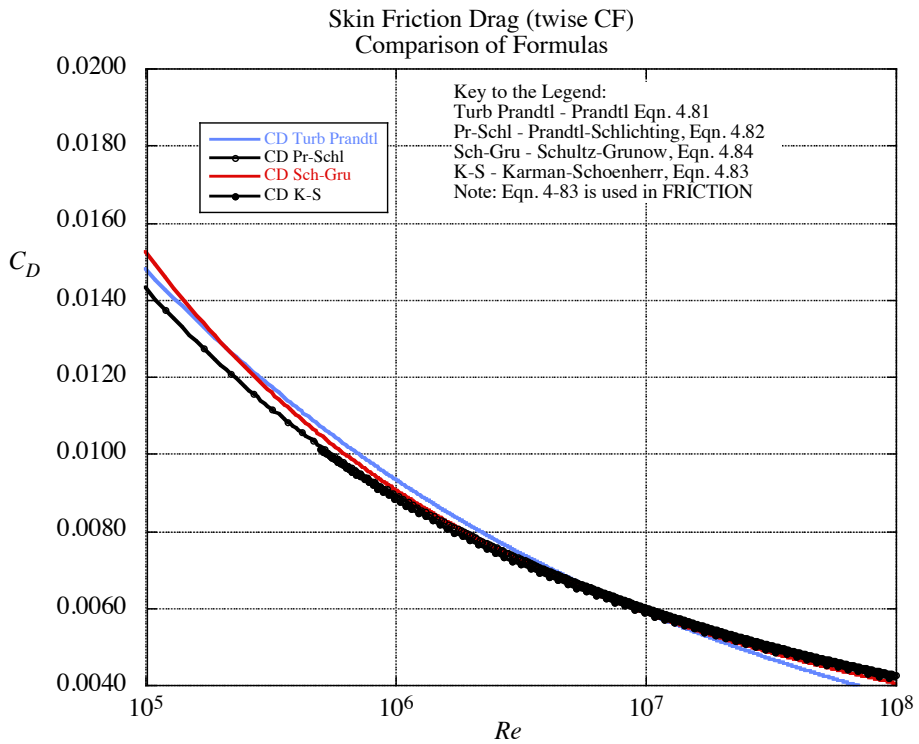

Figure C18-4. Skin friction drag coefficient formula comparisons with a "normal" scale.

Just to complete the presentation I'll include the figure with the curve often used to show a transition from the laminar to the turbulent results. B&C Fig. 4.19. This was interesting to me because there's no analytic solution for the intersection of the laminar curve and the transition curve. As is usually the case, Fig. B&C 4.19 shows a little overlap for the curve. This turns out to be a good problem in root finding using numerical methods to find the intersection. A while back I fired up my old modified regula falsi routine (essentially out of Conte's book) to find the intersection. The resulting curve is shown here as Figure C18-5. This would be a good problem to solve for aerospace students taking a computational methods class.
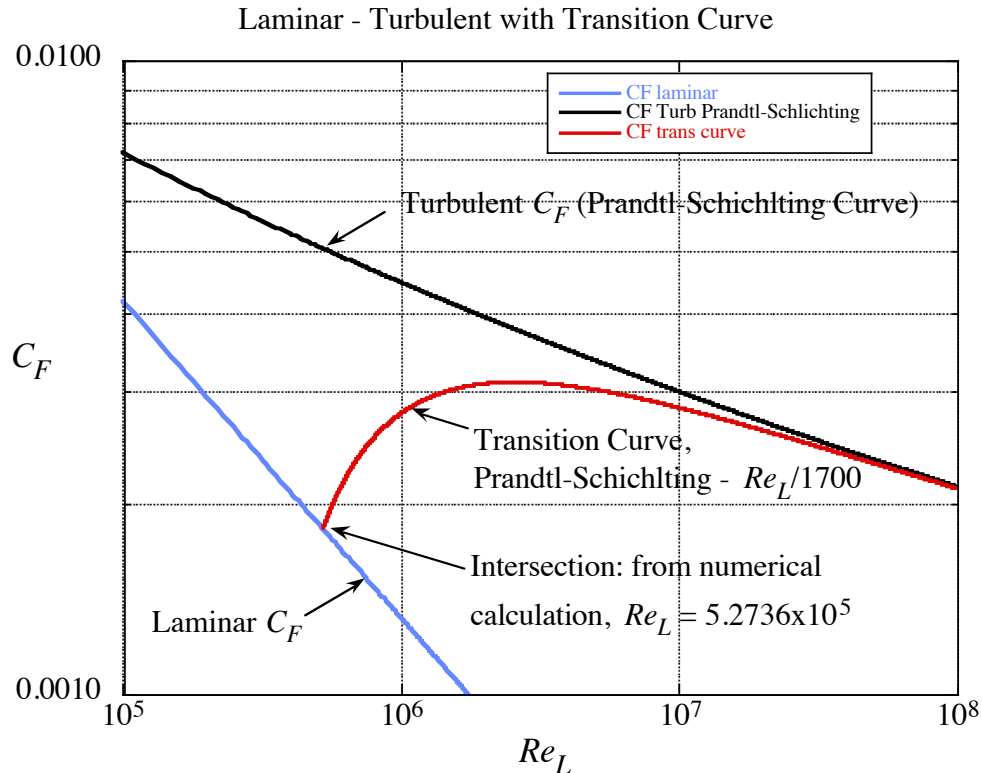

Figure C18-5. Skin friction drag coefficients showing the intersection between the laminar and transition curve.

The key points in this section are the "cross-over" of formulas at low Reynolds numbers, the performance of Prandtl's turbulent skin friction formula, which doesn't look that bad to me, and the application of numerical methods to find the intersection of two curves.

High speed skin friction drag trends with Mach number.

In working on the Hypersonics section of the Configuration Aerodynamics class notes I made some calculations to show the effect of Mach number on skin friction. I used my FRICTION code that employs the van Driest II method. In addition to the normal adiabatic wall cases I included cold and hot wall temperature cases. I made the calculations for Mach numbers to Mach 10. Somewhat surprisingly the predicted skin friction had one case "cross over" another at about $M = 6.5$. Admittedly a Mach numbers this high may be extreme for this method. The results are shown in Fig. C18-6.

Mach Number and Wall Temperature Ratio Effects
for Total Flat Plate Skin Friction using the Van Driest II Model

$T_w/T_e = 0.25$

$Re = 50$ million

$T_w/T_e = 1$

C_F
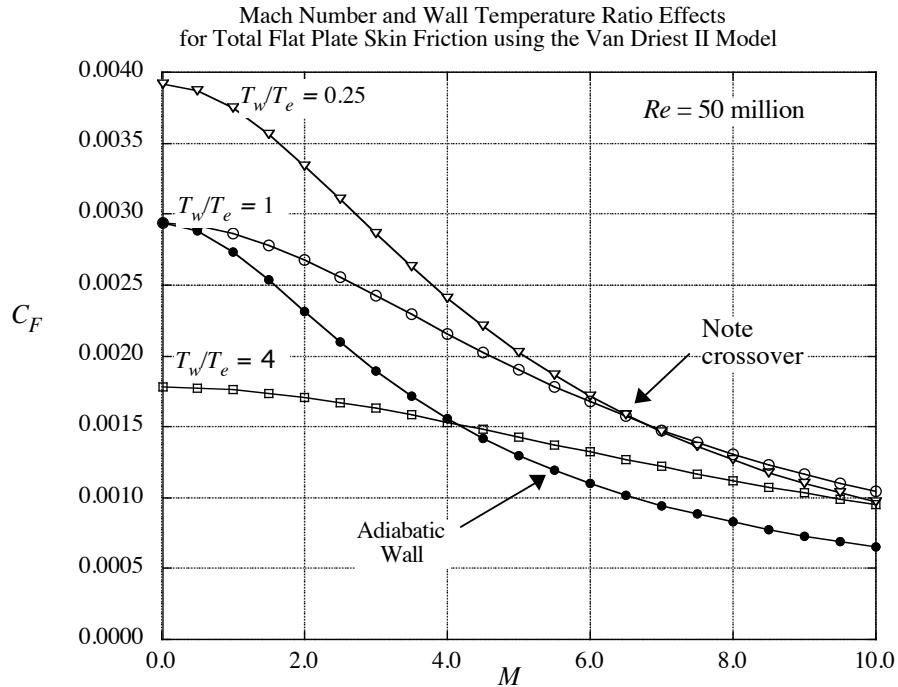
$T_w/T_e = 4$

Note
crossover

Adiabatic
Wall

M

Figure C18-6. Mach number and wall temperature effects on skin friction.

This method was selected based on the recommendation of E.J. Hopkins and M. Inouye, contained in "An Evaluation of Theories for Predicting Turbulent Skin Friction and Heat Transfer on Flat Plates at Supersonic and Hypersonic Mach Numbers," *AIAA J.*, Vol. 9, No. 6, June 1971, pp. 993-1003. The particular algorithm is taken from NASA TN D-6945, "Charts for Predicting Turbulent Skin Friction From the Van Driest Method (II)," also by E.J. Hopkins, and dated October 1972. He presented calculations up to $M = 10$, but didn't plot the results in this format. Admittedly I have a bias for using this method, having had the opportunity to take some turbulent boundary layer classes from Van Driest. We repeated his 1951 paper. That was a workout!

This case is not quite the same as the results shown in the classic book by Tuncer Cebeci and AMO Smith, *Analysis of Turbulent Boundary Layers*, Academic Press, 1974. However, looking at a result from their book suggests that the result I got is correct. I'm including their figure here as Figure C18-7.
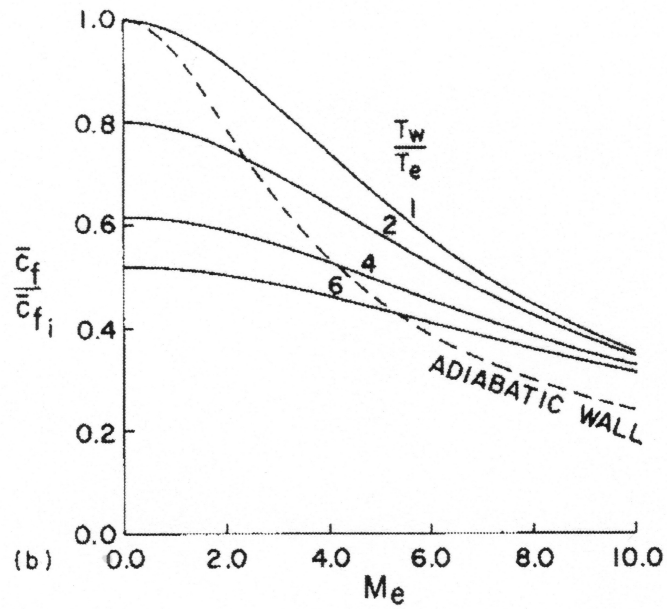
Figure C18-7. Similar results from Cebeci and Smith, their figure 5.10.

Cebeci repeated the figure in a later book with Bradshaw that included some additional comments, *Physical and Computational Aspects of Convective Heat Transfer*, Springer-Verlag, 1984. There it is Figure 11-3 and the Reynolds number is cited as 10 million. They also discuss a more general form of Van Driest's method, that Bradshaw terms Van Driest III. See Bradshaw, "An improved Van Driest skin-friction formula for compressible turbulent flows," *AIAA J.*, Vol. 15, No. 2, pp. 212, 1977.

The main point of this section was to point out the "crossover" seen in Fig. C18-6.