C16 The difference in wave drag between the Kármán Ogive and the Sears-Haack body

It is worth pointing out the minimum wave drag values for a couple of minimum wave drag axisymmetric bodies. The fineness ratio (*l*/*d*) plays a key role reducing the drag. Here we write the wave drag coefficients for three different cases: the Sears-Haack minimum wave drag for a given volume and length, the minimum wave drag for a specified maximum cross-sectional area, and the minimum wave drag for the Kármán ogive:

$$C_{D_{S-H}}\left(\frac{l}{d}\right)^2 = \frac{9\pi^2}{8} = 11.1 \qquad \text{(C16-1)}$$

$$C_{D_{l-S\max}}\left(\frac{l}{d}\right)^2 = \pi^2 = 9.87 \qquad \text{(C16-2)}$$

$$C_{D_{K-O}}\left(\frac{l}{d}\right)^2 = 1 \qquad \text{(C16-3)}$$

Observe that the minimum wave drag body for a closed body with a given max cross-sectional area is 11% less than the minimum wave drag body of a given volume with the same max cross-sectional area. The derivations of minimum wave drag axisymmetric body shapes for these and other constraints were given in NACA TN 2550 by M.C. Adams.[1] Read my Configuration Aerodynamics Supersonic notes for more information and figures showing the body shapes. Here the wave drag coefficients are based on the maximum cross-sectional area. The point is the amazingly large difference in values not emphasized in typical textbooks. I think this deserves to be considered a curiosity.

This also shows that ignoring base drag (reasonable if the base is filled with a jet exhaust), the drag is much lower when the base is open. In fact, the minimum wave drag is very nearly a linear function of the ratio of the base are to the max cross-sectional area (see Adam's Fig. 3):

$$C_{D_{0wave}}\left(\frac{l}{d}\right)^2 = \pi^2 - \left(\pi^2 - 1\right)\frac{S_B}{S_{\max}} \qquad \text{(C16-4)}$$

The minimum drag for a body of revolution with a constant cross section placed between the front and back of a body with a "fairing" to close the shape in the front and back has been given by Heaslet and Lomax, see Section Part D section 16.[2] To be more complete we cite another study. A key assumption associated with the minimum drag shapes described above is that the rate of change of the cross sectional area at the base is zero. Harder and Rennemann[3] found minimum drag shapes that have slightly less drag than Adams found by relaxing this requirement. They present results for the case of given volume.

Also, Krasnov[4] points out that using a simple cone forebody has a drag of about twice the value of the optimum forebody value for a fineness ratio of 3. However, a tangent ogive only as a drag about 7% higher than the optimum value.

This leads to some comments on minimum drag bodies of revolution including base drag. This is a little more difficult because there are no good simple formulas for supersonic base drag. However, the problem has been addressed. A fellow Hokie, Frankie Moore, working at the Navy's lab at Dahlgren, Virginia, studied this problem. It is important for gun projectiles. He describes the work in his book.[5] It turns out that the base diameter should be about 70% of the maximum diameter. Figure C16-1 Shows the details of the shape. References for the detailed work leading to this shape are cited in his references. Moore's book also discusses various base drag estimates.

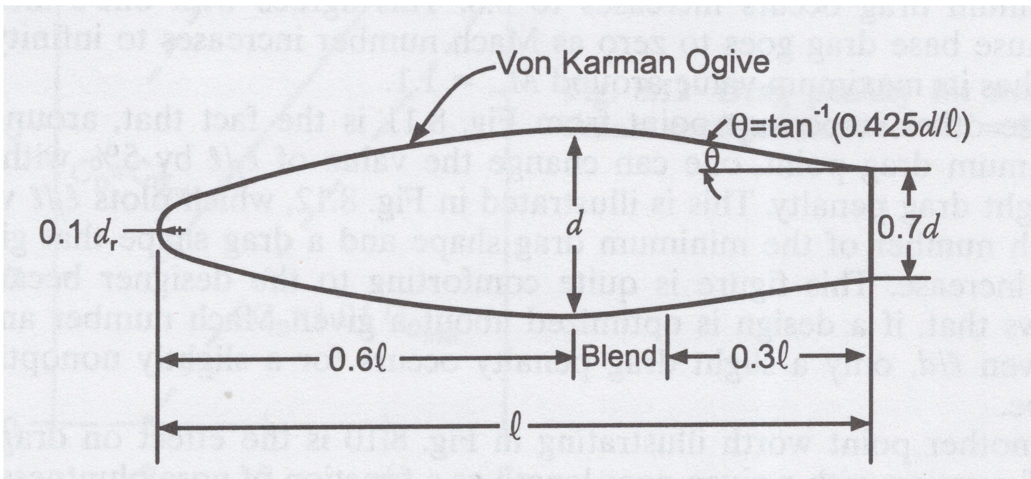

Figure C16-1 Moore's approximate minimum drag projectile.

The key takeaway here is that the wave drag of a von Kármán ogive is only about 9% of the wave drag of the famous Sears-Haack body.

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[1] M.C. Adams, "Determination of Shapes of Boattail Bodies of Revolution for Minimum Wave Drag," NACA TN 2550, Nov. 1951

[2] Max A. Heaslet and Harvard Lomax, "Supersonic and Transonic Small Perturbation Theory," in *General Theory of High Speed Aerodynamics*, W.R. Sear, Ed., Princeton University Press, Princeton, 1954, pg 240-249.

[3] Keith C. Harder and Conrad Rennemann, Jr., "On Boattail Bodies of Revolution Having Minimum Wave Drag," NACA R 1271, 1956.

[4] N. F. Krasnov, *Aerodynamics of Bodies of Revolution*, American Elsevier Publishing, New York, 1970, pp. pp. 376-378.

[5] Frank G. Moore, *Approximate Methods for Weapon Aerodynamics*, AIAA Progress in Astronautics and Aeronautics series, Vol. 186, 2000. pp. 423-429.