Aerodynamic Calculation Methods for Programmable Calculators & Personal Computers

— with programs for the TI-59 —

PAK #2
BASIC GEOMETRY FOR AERODYNAMICS

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AEROCAL

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Preface

AEROCAL programs are intended to serve both students and practicing aerodynamicists. For students, they can serve an important role in supplementing theoretical analysis with the actual numerical results so important in developing engineering skills. In aerodynamics, it has been difficult for students to solve meaningful illustrative problems, and this difficulty can now be eliminated by using the new personal computing machines -- either programmable calculators or microcomputers. I have found that the results of numerical calculations inevitably provide a few surprises, which force the analyst to reexamine the theory, leading to a much deeper understanding. AEROCAL programs can thus be used to prevent the calculation from becoming an end in itself. Instead, efforts can be concentrated on the actual aerodynamic problems, with required calculations assuming their proper supporting role. Thus, the availability of the personal computing machines allows the student to gain an appreciation of the role of computational aerodynamic simulations, while developing an engineering attitude.

The second purpose of the work is to provide the practicing aerodynamicist with a readily accessible collection of algorithms designed for use on this class of machine. The availability of such a set of routines will eliminate the most tedious aspects of the software development process so that the code development time can be used to implement the user's unique requirements rather than wasting time creating the basic building blocks.

The material selected for inclusion is, of course, not intended to replace the large computational aerodynamics programs. Instead, it allows students to become familiar with an important part of the set of standard aerodynamic methods representative of those required in aerodynamics. To the experienced user, these methods should be extremely useful, providing results which are more than adequate for a variety of jobs.

The material is organized in workbook fashion, with each program being essentially independent of the others. An example of the style that we intend to follow is found in the IBM SSP or other software package user's manuals. The addition of some examples for each program allows the user to check that the program is properly executing on his own machine.

The choice of the TI59 format for the programs is one of convenience only. Program instructions are similar for other calculators and an Appendix is included to describe the listing nomenclature. Using this information, conversion to other instruction sets should be relatively simple. Microcomputers will typically have more advanced instruction sets, such as BASIC. The information provided in the method description is easily used to write a set of BASIC instructions.

The author acknowledges the contributions of the many aerodynamicists and research scientists who have developed the basic material, which forms the basis for these software paks and with whom he has held discussions on the relative merits of particular methods for performing various aerodynamics calculations.

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Huntington, New York
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About the Author

W. H. Mason has spent more than ten years developing and applying computational aerodynamics methodology to transonic and supersonic aircraft design. This work required the use of the full range of computer codes presently used in the industry, so that the author has an unusually broad base of experience to draw upon. He obtained the B.S., M.S. and Ph.D. degrees in Aerospace Engineering at Virginia Polytechnic Institute and is presently employed as a Senior Engineer in the Aerodynamics Section of Grumman Aerospace Corporation. Dr. Mason is a registered Professional Engineer in New York State.

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2.0 INTRODUCTION

Aerodynamic design and analysis and geometry are very closely connected. Although this is obviously true, and no one would disagree, the degree to which geometry aspects of aerodynamics require attention and time would be surprising to those not actively involved in design and analysis. The author's experience is that aero design and analysis is generally thought to mean flow-field methodology. However, the detailed prescription of the geometric shape is the means through which vehicle performance is achieved and, thus, geometry is a very important part of aerodynamics.

The basic elements of geometry required for aerodynamics are the analysis of the planform and the specification of aerofoil sections and fuselage shapes. Methods for each of these elements are provided here. The planform analysis information is sufficient to provide all the geometric parameters normally required to describe the planform. The term "mean aerodynamic chord" has been used in place of "mean geometric chord," which is considered more correct but is not widely used. The basic parameters required in prescribing bodies of revolution are defined and a number of bodies used for supersonic vehicles are given. Finally, details of the useful NACA airfoils are given. The wide flexibility of shapes available from the analytic definitions provide an important base of information. These foils are still useful, either as defined or as baseline airfoils for modification.

The methods are presented in a standard format with the following information:

- o Title
- o Description of what the method does
- o References
- o Detailed outline of the method and listing of the equations required
- o User instructions
- o Sample case
- o Program description
- o Program listing.

The programs are written in the most direct sequence of instructions possible in order to make the study of the programs as simple as possible. This allows the user to incorporate modifications to the programs or convert them to other systems without difficulty. The use of the TI59 instruction set is purely a convenient selection. These routines will work on a number of other calculators, as well as the emerging class of microcomputers. For those readers not familiar with the details of the TI59 instruction set, a description is included in Appendix A. This will allow the non-TI59 user to convert the codes to his own instruction set with ease.

The routines often make use of the printer. The author has found the printer to be much more valuable in program development than in program execution. Nevertheless, several programs do provide printed results. A description of the printed output is included below the user instructions for each program.

2.1 PLANFORM ANALYSIS

This program computes both local and integral properties of a symmetrical wing planform. An additional short program is provided that gives the planform area for a complete configuration.

The local values provided are the leading and trailing edge locations $X_{LE}(y)$ and $X_{TE}(y)$, the local chord $C(y)$, and the leading and trailing edge sweep angles: $\Lambda_{LE}(y)$ and $\Lambda_{TE}(y)$. Sketch A illustrates the standard notation employed.

The integral properties computed are:

1. The planform area,

$$S = 2 \int_0^{b/2} C(y)\, dy$$

2. The mean aerodynamic chord:

$$\bar{c} = \frac{2}{S} \int_0^{b/2} C^2(y)\, dy$$

3. The X position of the centroid of area:

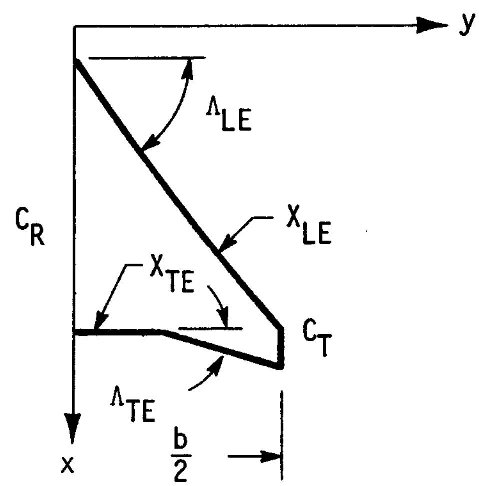
$$X_{CEN} = \frac{2}{S} \int_0^{b/2} C(y) \left(X_{LE}(y) + \frac{C(y)}{2} \right) dy$$

4. The spanwise position of $\bar{c}$:

$$Y_{MAC} = \frac{2}{S} \int_0^{b/2} y \cdot C(y)\, dy$$

5. The leading edge location of $\bar{c}$:

$$X_{LE_{MAC}} = \frac{2}{S} \int_0^{b/2} X_{LE}(y) \cdot C(y)\, dy$$



SKETCH A

In addition, the following derived quantities are of interest:

Aspect Ratio: $AR = \frac{b^2}{S_{REF}}$

Average Chord: $C_A = \frac{S_{REF}}{b}$

Taper Ratio: $\lambda = \frac{C_T}{C_R}$

Note that S_{REF} is usually chosen to be equal to the area of a basic reference trapezoidal planform and, thus, the actual planform area S may not equal S_{REF}.

METHOD OF SOLUTION

The leading and trailing edges are defined by a number of straight line segments, where the linear interpolation formula

$$f = f_{i-1} + \underbrace{\left[\frac{f_i - f_{i-1}}{x_i - x_{i-1}}\right]}_{\tan \Lambda} \left(x - x_{i-1}\right)$$

is used. The trapezoidal rule is then used to find the integral properties:

$$I = \int_0^{b/2} f\, dy = \Delta y \left[\frac{f_0}{2} + f_1 + f_2 + \ldots + f_{N-1} + \frac{f_N}{2}\right]$$

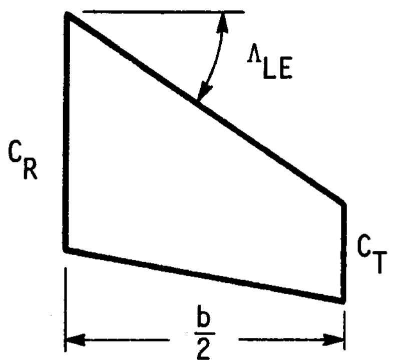
THE CENTROID OF TWO AREAS

If the combined centroid of a multiple surface system is desired, the following formulas are useful:

$$S\bar{x} = S_1\, \bar{x}_1 + S_2\, \bar{x}_2$$

$$S\bar{y} = S_1\, \bar{y}_1 + S_2\, \bar{y}_2$$

For standard trapezoidal wings it is convenient to collect the following formulas:



Sketch B

$$X_{LE}(y) = X_{LE_0} + \tan \Lambda_{LE} \cdot y$$

$$X_{TE}(y) = X_{TE_0} + \tan \Lambda_{TE} \cdot y$$

$$\frac{C}{C_R}(y) = 1 - (1 - \lambda)\,\eta$$

where $y = \frac{b}{2}\eta$

$$\eta = \frac{y}{b/2}$$

$$\tan \Lambda_n = \tan \Lambda_m - \frac{4}{AR}\left[(n-m)\left(\frac{1-\lambda}{1+\lambda}\right)\right]$$

where n, m are fractions of the local chord.

$$\text{Taper ratio, } \lambda = \frac{C_T}{C_R}$$

$$S = \frac{b}{2} \cdot C_R\,(1 + \lambda)$$

$$C_{AVE} = \frac{S}{b}$$

$$\frac{\bar{c}}{C_R} = \frac{2}{3} \cdot \left(\frac{1 + \lambda + \lambda^2}{1 + \lambda}\right)$$

$$AR = \frac{b^2}{S} = \frac{b/2}{C_R} \cdot \left(\frac{4}{1+\lambda}\right)$$

$$y_{MAC} = \frac{b}{6} \cdot \left(\frac{1+2\lambda}{1+\lambda}\right)$$

$$\frac{X_{LE_{MAC}}}{C_R} = \frac{X_{LE_0}}{C_R} + \left(\frac{1 + 2\lambda}{12}\right) \cdot AR \cdot \tan \Lambda_{LE}$$

$$X_{CEN} = X_{LE_{MAC}} + \frac{\bar{c}}{2}$$

USER INSTRUCTIONS -- PROGRAM 2.1

STEP	ENTER	PRESS	DISPLAY
Define the planform			
A. y_{LE}	y_{LE_1}	STO 11	
	y_{LE_2}	STO 12	
	⋮	⋮	
	y_{LE_N}	STO 10+N	
B. X_{LE}	Same as A, N_{MAX} = 10 on LE, = 9 on TE	STO 21 … 20+N	
C. Y_{TE}		STO 31 … 30+N	
D. X_{TE}		STO 41 … 40+N	
<u>Local Properties</u>			
1. Initialize	10	A	
2. y-station	y	SBR $\overline{X}$	C(y)
NOTE: Repeat 2. for each y station.	-	RCL 52	X_{LE} (y)
		53	X_{TE} (y)
		54	C (y)
		40	Λ_{LE} (y)
		RCL 50	Λ_{TE} (y)
<u>Integral Properties</u>			
1. Enter number of strips for integration.	M	A	
2. Enter number of points defining L.E.	N_{LE}	B	AR
NOTE: M = 10 is a typical value and the calculation takes about 2 minutes.		RCL 55	$\frac{S}{\overline{c}}$
		56	
		57	X_{CEN}
		58	y_{MAC}
		RCL 59	$X_{LE_{MAC}}$

If the printer is employed, unlabeled output is generated in the following order:

i) $\overline{c}$, ii) X_{CEN}, iii) y_{MAC}, iv) $X_{LE_{MAC}}$, v) S, vi) AR.

SAMPLE CASE

Input:

		STO		STO
y_{LE_1}	= 0.000	11	Y_{TE_1} = 0.000	31
y_{LE_2}	= 6.462	12	y_{TE_2} = 8.000	32
y_{LE_3}	= 14.697	13	y_{TE_3} = 14.697	33
X_{LE_1}	= 0.000	21	X_{TE_1} = 23.840	41
X_{LE_2}	= 13.858	22	X_{TE_2} = 25.447	42
X_{LE_3}	= 26.538	23	X_{TE_3} = 29.805	43

M = 20

N_{LE} = 3

Results:

AR = 2.525

S = 342.2

$\overline{c}$ = 14.75

X_{CEN} = 17.81

y_{MAC} = 5.186

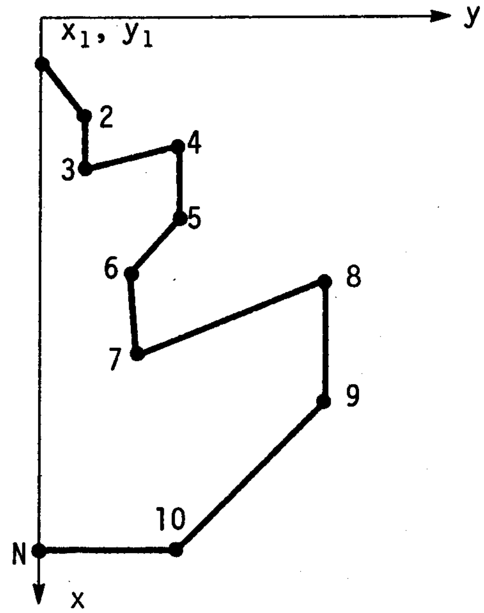
$X_{LE_{MAC}}$ = 10.44

DETAILS OF PROGRAM 2.1

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-35	This subroutine computes the local leading and trailing edge values and the chord.	0	M - number of spanwise strips
		1	12
		2	11
36-109	The linear interpolation routine for the leading edge.	3	22
		4	21 counters in
110-183	The linear interpolation routine for the trailing edge.	5	32 linear interpolation
		6	31 routines
		7	42
184-233	Initialization of linear interpolation routines and integral sum registers.	8	41
		9	Δy, AR
		10	b/2
234-360	The main routine; the step width Δy is computed and integrals are computed, and the final values are stored and printed out.	11-20	y_{LE}
		21-30	X_{LE}
		31-39	Y_{TE}
		40	Λ_{LE}
361-408	The summation subroutine for the integrals over the interior portion of the interval.	41-49	X_{TE}
		50	Λ_{TE}
		51	y
409-470	The summation subroutine for the integrals over the ends of the interval.	52	$X_{LE}(y)$
		53	$X_{TE}(y)$
		54	C(y)
		55	I_1 (s)
		56	I_2 ($\bar{c}$)
		57	I_3 (X_{CEN})
		58	I_4 (y_{MAC})
		59	I_5 ($X_{LE_{MAC}}$)

A SHORT PROGRAM FOR THE RAPID CALCULATION OF TOTAL PLANFORM AREA

By defining each point around the planform, the total area can be computed rapidly from:



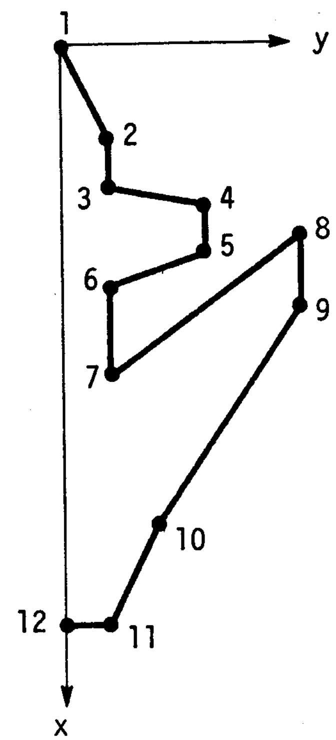
$$S = \sum_{K=1}^{N} \left(y_{K+1} + y_K\right) \left(x_{K+1} - x_K\right)$$

At K=N, y_{K+1}, X_{K+1} refer to the initial points y_1, x_1. For normal planforms, $y_{N+1} = y_1 = 0$, so that the summation can be terminated at N-1. This formula assumes planform symmetry and provides the total planform area with only one side of the planform computed.

USER INSTRUCTIONS -- PROGRAM 2.1A

STEP	ENTER	PRESS	DISPLAY
1. Initialize	-	A	
2. Enter first planform point.	$x_{K=1}$ $y_{K=1}$	x≷t B	
3. Enter the next pair location.	x_K y_K	x≷t C	
4. Repeat 3 until K=N or N-1.			S

SAMPLE PROBLEM: SHORT PROGRAM



ENTER:

	1	2	3	4	5	6	7	8	9	10	11	12
x	0	10	15	17	22	26	35	20	27	50	60	60
y	0	5	5	15	15	5	5	25	25	10	5	0
	B	C →									→	C

RESULT:

$S_{PLAN} = 1315.$

The author would like to acknowledge A. F. Vachris, Jr. for pointing this formula out to him. It is used extensively in computing cross-sectional areas during the determination of volumetric wave drag.

2.2 BODIES OF REVOLUTION

Bodies of revolution form the basis for a number of shapes used in aerodynamic design and are also often used in comparing computational methods. The bodies contained in this section are generally associated with supersonic aerodynamics. Each program is similar in input, output and storage.

(a) Summary of Relations

The body radius r is given as a function of x, $\frac{r}{\ell} = f\left(\frac{x}{\ell}\right)$. Once r is known, a number of other values characterizing the shape can be determined.

Cross-sectional area and derivatives are:

$$S(x) = \pi r^2$$

$$\frac{dS}{dx} = 2\pi r \frac{dr}{dx}$$

$$\frac{d^2S}{dx^2} = 2\pi \left[\left(\frac{dr}{dx}\right)^2 + r \frac{d^2r}{dx^2} \right]$$

Basic integrals are:

$$\text{Volume,} \quad V = \int_0^{\ell} S(x)\, dx$$

$$\text{Surface area,} \quad S_{WET} = 2\pi \int_0^{\ell} r(x)\, dx$$

$$\text{Length along contour,} \quad p(\bar{x}) = \int_0^{\bar{x}} \sqrt{1 + \left(\frac{dr}{dx}\right)^2}\, dx$$

Note that incremental values can be found by changing the lower limit of the integrals.

The local longitudinal radius of curvature is given by:

$$\frac{1}{R(x)} = \frac{\left| \dfrac{d^2r}{dx^2} \right|}{\left[1 + \left(\dfrac{dr}{dx}\right)^2 \right]^{3/2}}$$

Several simple shapes, in addition to those presented in detail, are also of interest:

Parabolic spindle: $\frac{r}{\ell} = 4 \frac{r_{mid}}{\ell} \cdot \frac{x}{\ell} \left(1 - \frac{x}{\ell}\right)$

Ellipsoid of revolution: $\frac{r}{\ell} = 2 \cdot \frac{r_{mid}}{\ell} \sqrt{\frac{x}{\ell} \left(1 - \frac{x}{\ell}\right)}$

Power law: $\frac{r}{\ell} = \frac{r_o}{\ell} \left(\frac{x}{X_N}\right)^n$

Where X_N is the nose length and r_o is the radius at $x = X_N$. The nose is blunt for $0 < n < 1$.

Another common shape is the spherical nose cap, and is discussed in detail in the reference by Krasnov.

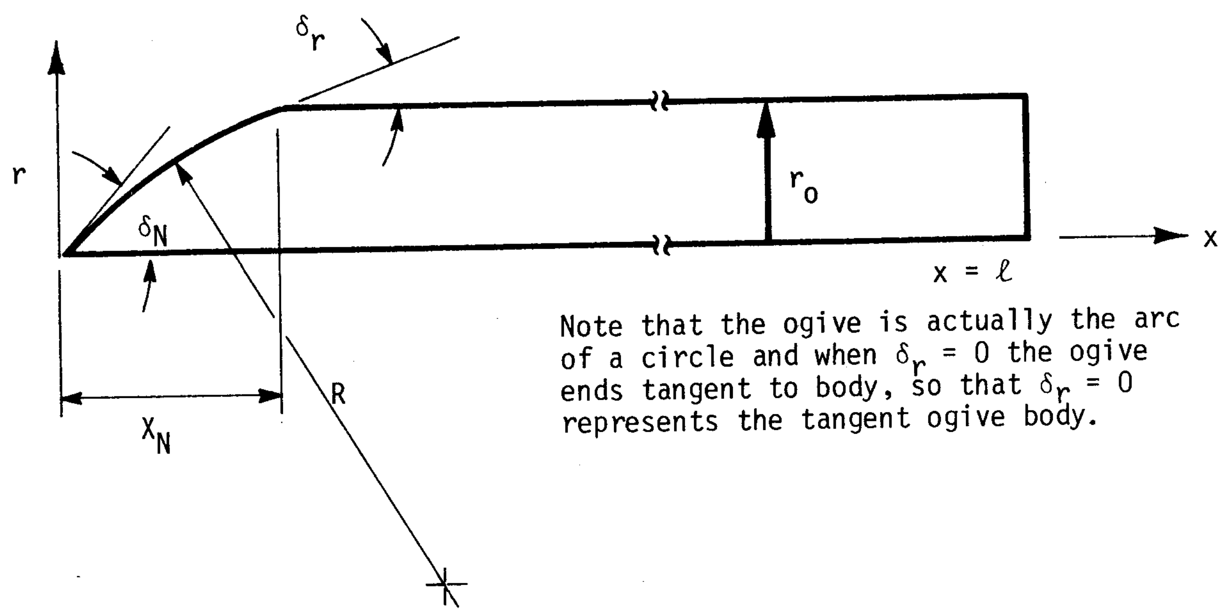
References that discuss the geometry of bodies of revolution are:

Krasnov, N. F., Aerodynamics of Bodies of Revolution, edited and annotated by D. N. Morris, American Elsevier, New York, 1970.

Handbook of Supersonic Aerodynamics, Volume 3, Section 8, Bodies of Revolution, NAVWEPS Report 1488, October 1961.

(b) Tangent/Secant Ogives

This program generates the radius, cross-sectional area, and their first and second derivatives for tangent and secant ogives. A discussion of these shapes can be found in NAVWEPS Report 1488, Volume 3, Section 8, pp. 58-61 and pp. 222-223. The nomenclature is illustrated in the following sketch.



Note that the ogive is actually the arc of a circle and when $\delta_r = 0$ the ogive ends tangent to body, so that $\delta_r = 0$ represents the tangent ogive body.

If $\delta_r = \delta_N$, the cone-cylinder case is recovered (in the program, $\delta_r = \delta_N - .1 \times 10^5$ is actually required as an input), and if $\delta_r = 0$, $\delta_N = 90°$, a spherical cap case is obtained.

The expression for the radius r is determined using three basic constants for a particular case:

$$A = \frac{r_o}{\ell}\left(\frac{\cos\delta_N}{\cos\delta_r - \cos\delta_N}\right)$$

$$B = 2\,\frac{r_o}{\ell}\left(\frac{\sin\delta_N}{\cos\delta_r - \cos\delta_N}\right)$$

and

$$C = \frac{r_o}{\ell}$$

The radius is then given by:

$$\frac{r}{\ell} = \sqrt{A^2 + B\left(\frac{x}{\ell}\right) - \left(\frac{x}{\ell}\right)^2} - A \qquad 0 < \frac{x}{\ell} < \frac{X_N}{\ell}$$

$$= C \qquad \frac{X_N}{\ell} < \frac{x}{\ell} < 1$$

Where X_N is found as follows.

For a tangent ogive ($\delta_r = 0$), the ogive can be defined by specifying either X_N/r_o or δ_N. The other value can then be found using:

Given δ_N, $$\frac{X_N}{r_o} = \frac{\sin\delta_N}{1-\cos\delta_N}$$

Or given X_N/r_o, $$\delta_N = \cos^{-1}\left[\frac{(X_N/r_o)^2 - 1}{(X_N/r_o)^2 + 1}\right]$$

For the secant ogive, the simplest analytical procedure is to define the ogive in terms of δ_N and δ_r, and find X_N/ℓ from:

$$\frac{X_N}{\ell} = \frac{r_o}{\ell}\left[\frac{\sin\delta_N - \sin\delta_r}{\cos\delta_r - \cos\delta_N}\right]$$

If X_N/ℓ is not satisfactory, δ_N and δ_r can be adjusted by trial and error to obtain the desired nose length. The program is set up to handle this process quite simply.

The first and second derivatives are then given by:

$$\frac{d(r/\ell)}{d(x/\ell)} = \frac{B - 2\,(x/\ell)}{2\left[\frac{r}{\ell} - A\right]}$$

and

$$\frac{d^2(r/\ell)}{d(x/\ell)^2} = -\frac{\left[B - 2\,(x/\ell)\right]^2}{4\left[(r/\ell) - A\right]^3} - \frac{1}{\left[r/\ell - A\right]}$$

The relationships between radius and area derivatives given in Section 2.2 (a) are then used to complete the calculation.

USER INSTRUCTIONS -- PROGRAM 2.2(b)

STEP	ENTER	PRESS	DISPLAY
1. Input radius	r_o/ℓ	A	
TANGENT OGIVE			
2T. Input nose angle	δ_N	B	X_N/ℓ
or			
3T. Input nose length	X_N/ℓ	C	δ_N
SECANT OGIVE			
2S. Input nose angle	δ_N	B'	
3S. Input intersection angle	δ_r	C'	X_N/ℓ
4. Input x station	x/ℓ	D	r/ℓ
		RCL 12	$(r/\ell)'$
		RCL 13	$(r/\ell)''$
		RCL 14	S/ℓ^2
Repeat step 4. for each value of x/ℓ desired.		RCL 15	$(S/\ell^2)'$
		RCL 16	$(S/\ell^2)''$

If the printer is employed, unlabeled output is generated in the following order: Steps 1, 2, 3 input echo, then after Step 4: x/ℓ, r/ℓ, $(r/\ell)'$, and $(r/\ell)''$.

SAMPLE CASE

TANGENT OGIVE: $\frac{r_o}{\ell} = .05,\ \delta_N = 20°$

$$\rightarrow \frac{X_N}{\ell} = .28356$$

x/ℓ	r/ℓ	$(r/\ell)'$	$(r/\ell)''$	(S/ℓ^2)	$(S/\ell^2)'$	$(S/\ell^2)''$
0.00	0.0000	0.3640	-1.4536	0.0000	0.0000	0.8324
0.06	0.0193	0.2800	-1.3508	0.0012	0.0339	0.3290
0.12	0.0337	0.2012	-1.2802	0.0036	0.0426	-0.0167
0.18	0.0435	0.1259	-1.2349	0.0059	0.0344	-0.2380
0.24	0.0489	0.0526	-1.2112	0.0075	0.0162	-0.3544
0.30	0.0500	0.0000	0.0000	0.0079	0.0000	0.0000

To test $\frac{X_N}{\ell}$; $\frac{r_o}{\ell} = .05,\ \frac{X_N}{\ell} = .20 \rightarrow \delta_N = 28.0725°$

SECANT OGIVE: $\frac{r_o}{\ell} = .05 \quad \left\{ \begin{array}{l} \delta_N = 20° \\ \delta_r = 10° \end{array} \right\} \rightarrow \frac{X_N}{\ell} = .1866$

x/ℓ	r/ℓ	$(r/\ell)'$	$(r/\ell)''$	(S/ℓ^2)	$(S/\ell^2)'$	$(S/\ell^2)''$
0.00	0.0000	0.3640	-1.0874	0.0000	0.0000	0.8324
0.06	0.0199	0.3006	-1.0273	0.0012	0.0376	0.4392
0.12	0.0361	0.2404	-0.9816	0.0041	0.0546	0.1402
0.18	0.0488	0.1826	-0.9478	0.0075	0.0560	-0.0813
0.24	0.0500	0.0000	0.0000	0.0079	0.0000	0.0000

DETAILS OF PROGRAM 2.2(b)

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-17	Routine D, the main program. Starts by deciding if x is greater or less than X_N.	6	δ_r
		7	X_N/ℓ
18-46	Returns values for $x > X_N$.	8	δ_N
47-167	Computes and stores values for $x < X_N$.	9	r_o/ℓ
		10	x/ℓ
168-209	Routine π to compute and store constants A, B and $\cos \delta_r - \cos \delta_N$.	11	r
		12	r'
210-220	Stores r_o/ℓ.	13	r''
221-249	Stores δ_N and computes X_N/ℓ.	14	S
250-289	Stores X_N and computes δ_N; calls routine π.	15	S'
290-297	Store δ_N.	16	S''
298-345	Store δ_r and compute X_N/ℓ; calls routine π.	17	A
		18	B
		19	r_o/X_N
		20	$\cos \delta_r - \cos \delta_N$
		21	$\sqrt{A^2 + B x - x^2}$

NOTE: Registers 0-5 are not used to allow modification to use ML-09 to compute integral quantities such as volume and arc length using this routine.

2.2 (c) The von Karman Ogive

The program generates the ordinates and derivatives of the radius and cross-sectional area distribution for the von Karman Ogive. This is the shape that produces minimum wave drag for a specified base area and length, according to slender body theory. This ogive has a very slightly blunted nose, and is described by Ashley and Landahl, Aerodynamics of Wings and Bodies, Addison-Wesley, 1965, pp. 178-181.

In this case, it is convenient to work with the cross-sectional area and a new independent variable:

$$\theta = \cos^{-1}\left[2\left(\frac{x}{X_N}\right) - 1\right]$$

or

$$\frac{x}{X_N} = \frac{1}{2}\left(1 + \cos\theta\right)$$

where the nose is at $\theta = \pi$, and the base is located at $\theta = 0$.

Here we use X_N to denote the "nose length" or length of the ogive, and allow this shape to be part of an ogive-cylinder geometry.

The shape is then given as:

$$\frac{S(x)}{\ell^2} = \frac{S_B}{\ell^2} \cdot \left[1 - \frac{\theta}{\pi} + \frac{1}{2\pi} \cdot \sin 2\theta\right]$$

and

$$\frac{r}{\ell} = \sqrt{\frac{S/\ell^2}{\pi}}$$

where S_B is the prescribed base area and ℓ is the total length.

Defining $\bar{S} = \frac{S}{\ell^2}$, $\bar{x} = \frac{x}{\ell}$

We have $$\frac{d\bar{S}}{d\theta} = -\frac{\bar{S}_B}{\pi} \cdot \left[1 - \cos 2\theta\right]$$

$$\frac{d^2\bar{S}}{d\theta^2} = -\frac{2}{\pi} \cdot \bar{S}_B \cdot \sin 2\theta$$

and $$\frac{d\bar{S}}{dx} = S' = \frac{4}{\pi} \cdot \left(\frac{\ell}{X_N}\right) \bar{S}_B \sin \theta$$

$$\frac{d^2\bar{S}}{dx^2} = \bar{S}'' = -\frac{8}{\pi} \cdot \left(\frac{\ell}{X_N}\right)^2 \frac{\bar{S}_B}{\tan \theta}$$

The radius derivatives are then computed by:

$$\frac{d\bar{r}}{dx} = \frac{\bar{S}'}{2\pi\bar{r}} \quad ; \quad \frac{d^2\bar{r}}{dx^2} = \frac{\bar{S}''}{2\pi\bar{r}} - \frac{\bar{r}'^2}{\bar{r}} \; .$$

The normal operation of the program is carried out specifying r_B/X_N and X_N/ℓ. For $X_N = \ell$, the program can also be run by specifying ℓ and S_B.

USER INSTRUCTIONS -- PROGRAM 2.2(c)

STEP	ENTER	PRESS	DISPLAY
BASIC MODE			
1. Input nose ratio	r_B/X_N	A	
2. If $X_N/\ell \neq 1$	X_N/ℓ	B	
SECONDARY MODE			
1S. Input length	ℓ	B'	
2S. Input base area	S_B	C'	
3. Input x station	x/ℓ	D	r/ℓ
		RCL 12	$(r/\ell)'$
Repeat Step 3. for each value of x/ℓ desired.		RCL 13	$(r/\ell)''$
		RCL 14	S/ℓ^2
		RCL 15	$(S/\ell^2)'$
		RCL 16	$(S/\ell^2)''$

If the printer is employed, unlabeled output is generated as follows: Steps 1 and 2; the input is echoed. At Step 3, x/ℓ is printed followed by x/X_N and $\bar{r}$, $\bar{r}'$, and $\bar{r}''$.

SAMPLE CASE

$$\frac{r_B}{X_N} = .1; \quad \frac{X_N}{\ell} = .40$$

x/ℓ	r/ℓ	$(r/\ell)'$	$(r/\ell)''$	(S/ℓ^2)	$(S/\ell^2)'$	$(S/\ell^2)''$
0.00	0.0000	0.0000*	∞	0.0000	0.0000	∞
0.10	0.0177	0.1247	- .4635	0.0010	0.0139	0.0462
0.20	0.0283	0.0900	- .2866	0.0025	0.0160	0.0000
0.30	0.0359	0.0615	- .3102	0.0040	0.0139	-0.0462
0.39	0.0399	0.0199	- .9817	0.0050	0.0050	-0.2434
0.41	0.0400	0.0000	0.0000	0.0050	0.0000	0.0000

* At $x/\ell = 0.0$, the slope is infinite (the nose is blunt). However, the program produces a zero value due to the particular combination of numerical operations.

DETAILS OF PROGRAM 2.2(c)

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-19	Routine D, the main program, starts by testing to see if x is greater or less than X_N.	6	X_N/ℓ
		7	X_N/r_B
20-43	Values for $x > X_N$ are computed.	8	S_B/ℓ^2
		9	θ
44-167	Values for $x < X_N$ are computed:	10	x/X_N
	46-87 S 88-95 r 96-114 S' 115-126 r' 127-146 S" 147-167 r"	11	r/ℓ
		12	$(r/\ell)'$
		13	$(r/\ell)''$
		14	S/ℓ^2
170-182	Stores r_B/X_N and sets X_N/ℓ = 1 and call routine π.	15	$(S/\ell^2)'$
183-190	Stores X_N/ℓ for $X_N/\ell \neq 1$ and calls routine π.	16	$(S/\ell^2)''$
191-206	Routine π. Compute S_B/ℓ^2 and store.	17	x/ℓ
		18	ℓ
207-216	Read in ℓ and set X_N/ℓ = 1.	19	S_B
217-249	Read in S_B and X_N/r_B and S_B/ℓ^2.	20	r_B

NOTE: Registers 0-5 are not used to allow modification to use ML-09 to compute integral quantities such as volume and arc length using this routine.

2.2 (d) The Sears-Haack Body

This program generates the radius and cross-sectional area and their first and second derivatives for the Sears-Haack Body, which is the minimum drag shape for a given length and volume according to slender body theory. This body is closed at both ends, is symmetric about the mid-point, and has slightly blunted ends. It is described in Ashley and Landahl, Aerodynamics of Wings and Bodies, Addison-Wesley, 1965, pp. 178-181.

Although the notation used in Section 2.2(c) for the von Karman Ogive Section could be used, it is more common to describe the Sears-Haack shape in the manner presented below. This form uses the fineness ratio, $f = \ell/d_{MAX}$ to scale the shape. However, it is important to realize that the Sears-Haack shape is the minimum drag body for a specified volume and length and not for a specified fineness ratio. The minimum drag body for a specified fineness ratio will be described in Section 2.2(e).

Defining

$$\zeta = 1 - 2\left(\frac{x}{\ell}\right),$$

the Sears-Haack body is defined as

$$\frac{r}{\ell} = \frac{1}{2f}\left(1 - \zeta^2\right)^{3/4} .$$

The derivatives are given by:

$$\frac{d(r/\ell)}{d(x/\ell)} = \frac{3\zeta}{1 - \zeta^2} \cdot \left(\frac{r}{\ell}\right)$$

and

$$\frac{d^2(r/\ell)}{d(x/\ell)^2} = -\left(\frac{1}{1 - \zeta^2}\right) \cdot \left[\zeta \frac{d(r/\ell)}{d(x/\ell)} + 6\left(\frac{r}{\ell}\right)\right].$$

The fineness ratio is related to the length and volume by:

$$f = \sqrt{\frac{3\pi^2}{64} \cdot \frac{\ell^3}{V}} .$$

In terms of f and either V or ℓ, the other value can be found from the following:

Given f and ℓ: $$V = \frac{3\pi^2}{64} \cdot \frac{\ell^3}{f^2}$$

And, given f and V: $$\ell = \left[V \cdot \frac{64}{3\pi^2} f^2\right]^{1/3}$$

The relationships between radius and area derivatives given in Section 2.2(a) are then used to complete the calculation.

USER INSTRUCTIONS -- PROGRAM 2.2(d)

STEP	ENTER	PRESS	DISPLAY
a. Given ℓ, V			
1a. Input ℓ	ℓ	B'	
2a. Input V	V	C'	f
b. Given f and ℓ or V			
1b. Input f	f	A	
2bi. Input ℓ	ℓ	B	V
OR			
2bii. Input V	V	C	ℓ
3. Input x station	$\frac{x}{\ell}$	D	r/ℓ
		RCL 12	$(r/\ell)'$
Repeat Step 3. for each value of x/ℓ desired.		RCL 13	$(r/\ell)''$
		RCL 14	S/ℓ^2
		RCL 15	$(S/\ell^2)'$
		RCL 16	$(S/\ell^2)''$

If the printer is employed, unlabeled output is generated as follows: Step and 2: the input is echoed and the display value printed. At Step 3, x/ℓ is printed followed by r/ℓ, $(r/\ell)'$, and $(r/\ell)''$.

SAMPLE CASE

$f = 7,\ \ell = 1.0 \rightarrow V = .00944$

x/ℓ	r/ℓ	$(r/\ell)'$	$(r/\ell)''$	S/ℓ^2	$(S/\ell^2)'$	$(S/\ell^2)''$
0.0	0.0000	1.?	∞	0.0000	0.0000	6.2832*
0.1	0.0332	0.2213	-1.0451	0.0035	0.0462	0.0898
0.2	0.0511	0.1437	-0.6139	0.0082	0.0462	-0.0673
0.3	0.0627	0.0895	-0.4903	0.0123	0.0353	-0.1427
0.4	0.0693	0.0433	-0.4420	0.0151	0.0188	-0.1806
0.5	0.0714	0.0000	-0.4286	0.0160	0.0000	-0.1923

* Note that at the nose, the slope is infinite and the numerical sequence causes the calculation to flash and produce ?. This also leads to incor value of $(S/\ell^2)''$, which should also be infinite.

To test i) given $f = 7,\ \ell = 5 \rightarrow V = 1.1802$

ii) given $f = 7,\ V = 2 \rightarrow \ell = 5.9611$

iii) given $\ell = 4,\ V = .8 \rightarrow f = 6.0837$

DETAILS OF PROGRAM 2.2(d)

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-124	Routine D, the main program: Starts by reading and storing x/ℓ.	6	V
		7	ℓ
7-16	ζ	8	ζ
17-36	r	9	f
37-45	S	10	x/ℓ
46-64	r'	11	r/ℓ
66-77	S'	12	$(r/\ell)'$
78-102	r"	13	$(r/\ell)''$
104-121	S"	14	S/ℓ^2
125-131	Read and store f.	15	$(S/\ell^2)'$
132-157	Read and store ℓ, compute V.	16	$(S/\ell^2)''$
158-189	Read and store V, compute ℓ.		
190-196	Read and store ℓ.		
197-223	Read and store V, compute f.		

NOTE: Registers 0-5 are not used to allow modification to use ML-05 to compute integrals.

2.2 (e) The Haack-Adams Bodies

This program computes the radius and cross-sectional area for each of the three classes of minimum drag bodies described by M. C. Adams in "Determination of Shapes of Boattail Bodies of Revolution for Minimum Wave Drag," NACA TN 2550, November 1951. These bodies correspond to the following cases:

I. Given length, base area, and contour passing through a specifically located radius.

II. Given length, base area, and maximum area.

III. Given length, base area, and volume.

In Case I, the specified radius will not necessarily be the maximum radius.

The notation used in TN 2550 is employed in the equations, leading to the following definitions:

$$S = 4\,\frac{\overline{S}(x)}{\ell^2}, \qquad B = 4\,\frac{S_{BASE}}{\ell^2}, \qquad A = 4\,\frac{S_A}{\ell^2}, \qquad V = 8\left(\frac{\overline{V}}{\ell^3}\right)$$

where $\overline{S}(x)$ is the area, S_A corresponds to either the specified area at a given location, or the maximum area, and $\overline{V}$ is the volume. The independent variable is defined with its origin at the body mid-point:

$$\zeta = 2\left(\frac{x}{\ell}\right) - 1$$

and the location of the specified radius (Case I) and maximum radius (Case II) is designated C and given in ζ coordinates. When referred to the x coordinate, this value is designated C_x.

The equations for each case can be written in a standard form:

<u>Case I</u> -- Given S_B, S_A, C_x:

$$\frac{\pi S}{B} = \left[\frac{\pi A}{B} - \cos^{-1}(-c)\right]\frac{\sqrt{1-\zeta^2}\,(1-c\zeta)}{\left[1-c^2\right]^{3/2}} + \frac{\sqrt{1-\zeta^2}\cdot(\zeta - c)}{\left[1-c^2\right]}$$

$$+\left[\frac{\pi A}{B} - \cos^{-1}(-c) - c\sqrt{1-c^2}\right]\frac{(\zeta - c)^2}{(1-c^2)^2}\,\ell n\,N + \cos^{-1}(-\zeta)$$

$$\text{where } N = \frac{1 - c\zeta - \sqrt{1-c^2}\,\sqrt{1-\zeta^2}}{|\zeta - c|}$$

Case II -- Given S_B, S_{MAX}:

First find the location of the maximum thickness from the implicit relation

$$f(c) = 0 = \frac{\pi A}{B} \cdot c - \sqrt{1 - c^2} - c \cdot \cos^{-1}(-c).$$

Use Newton's iteration,

$$c^{i+1} = c^i - \frac{f(c^i)}{f'(c^i)},$$

where $$f'(c) = \frac{\pi A}{B} - \cos^{-1}(-c).$$

An initial guess of c = 0 is sufficient to start the iteration. Given c, the relation is:

$$\frac{\pi S}{B} = \frac{\sqrt{1 - \zeta^2}}{c} + \frac{(\zeta - c)^2}{c\sqrt{1 - c^2}} \cdot \ell n\, N + \cos^{-1}(-\zeta)$$

where N is the same function as given in Case I.

Case III -- Given S_B and $\overline{V}$:

$$\frac{\pi S}{B} = \frac{8}{3}\left[\frac{V}{B} - 1\right]\left[1 - \zeta^2\right]^{3/2} + \zeta\sqrt{1 - \zeta^2} + \cos^{-1}(-\zeta).$$

The maximum thickness for this case is located at

$$e = \frac{1}{4\,(V/B - 1)},$$

and in x coordinates

$$e_x = \frac{1}{2}(1 + e).$$

Note that if S_B = 0, the Sears-Haack body is recovered.

In order to simplify the analysis, S_B is not allowed to be identically zero within the present program, but is reset to 1 X 10^{-7} for all cases.

USER INSTRUCTIONS -- PROGRAM 2.2(e)

STEP	ENTER	PRESS	DISPLAY
1. Input base area*.	$\overline{S}(\ell)/\ell^2$	A	
If Case I or II			
2A. Input max area (Case II), or specified area (Case I)	S_A/ℓ^2	B	$\frac{c_x}{\ell}$, the location of S_{MAX} for Case II.
If Case I			
2B. Input c, the location of the specified area.	c_x/ℓ	B'	
If Case III			
2C. Input volume.	$\overline{V}/\ell^3$	C	e_x, the position of max thickness.
3. Input x station:			
3A. Case Given			
I S_B, S(c), c	x/ℓ	D RCL 14	$r(x)/\ell$ $\overline{S}(x)/\ell^2$
3B. II S_B, S_{MAX}	x/ℓ	E RCL 14	$r(x)/\ell$ $\overline{S}(x)/\ell^2$
3C. III S_B, V	x/ℓ	E' RCL 14	$r(x)/\ell$ $\overline{S}(x)/\ell^2$
Repeat Step 3. for each value of x/ℓ desired.			

* If $S(\ell) = 0$, program resets it to 0.1×10^{-8}.

If the printer is employed, unlabeled output is generated. Each input is echoed and r/ℓ is printed. After B is pressed, the iteration history of c is output.

SAMPLE CASES

I. Given $\frac{S_B}{\ell^2} = .003927$, $\frac{S(c)}{\ell^2} = .0314159$, $\frac{c}{\ell} = \frac{2}{3}$

x/ℓ	r/ℓ
0.0	0.0000
0.2	0.0565
0.4	0.0867
0.6	0.1009
0.8	0.0812
1.0	0.0354

NOTE: This case corresponds to Figure 1a in TN 2550 with B/A = .125, c = 1/3.

II. Given $\frac{S_B}{\ell^2} = .0019635$, $\frac{S_{MAX}}{\ell^2} = .0314159$

$$\rightarrow \left.\frac{c}{\ell}\right|_{\text{MIN DRAG}} = .51027$$

NOTE: This case corresponds to Figure 1b in TN 2550 with B/A = 1/16.

x/ℓ	r/ℓ
0.0	0.0000
0.2	0.0627
0.4	0.0934
0.6	0.0954
0.8	0.0673
1.0	0.0250

III. Given $\frac{S_B}{\ell^2} = .0085312$, $\frac{V}{\ell^3} = .0109306$

$$\rightarrow \left.\left(\frac{x}{\ell}\right)\right|_{S_{MAX}} = 0.58$$

NOTE: This case corresponds to the f = 7 Haack-Adams body given in NASA TN D-3163, December 1965.

x/ℓ	r/ℓ
0.0	0.0000
0.2	0.0472
0.4	0.0663
0.6	0.0713
0.8	0.0646
1.0	0.0521

DETAILS OF PROGRAM 2.2(e)

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-20	Input S_B/ℓ^2 and save B (reset if S_B = 0).	6	V
		7	c (in ζ variable)
21-128	Input S and solve for c for S = S_{MAX} via Newton iteration.	8	B
		9	A
129-184	Routine to compute N for $\zeta \neq$ c.	10	x/ℓ
185-196	If ζ = c, reset N to 1. X 10^{-20}.	11	r/ℓ
		12	-
197-228	Read c in X variable and compute and store c in the ζ coordinate system.	13	-
		14	$\overline{S}/\ell$
229-268	Read V, store $\overline{V}$, compute e and e_x.	15	
		16	
269-364	Routine D, compute r and S for Case I.	17	e (for Case III, ζ variable).
365-381	Routine SUM, stores x/ℓ and computes and stores .	18	A/B
382-406	Routine STO; computes $S(x)/\ell^2$ from ($\pi S/B$), and computes r/ℓ from S/ℓ^2.	19	$\pi A/B$
		20	C_{OLD}; $\pi A/B - \cos^{-1}(-c)$
407-455	Routine E, computes r and S for Case II.	21	C_{NEW}; $\sqrt{1 - \zeta^2}$
		22	ζ
456-509	Routine E', computes r and S for Case III.	23	$\sqrt{1 - c^2}$
		24	$\cos^{-1}(-c)$
		25	N
		26	e_x, Case II, III

2.3 THE NACA AIRFOILS

The NACA airfoils were designed during the period from 1929 through 1947 under the direction of Eastman Jacobs at the NACA's Langley Field Facility. Most of the foils were based on simple geometrical descriptions of the section shape. Although a new generation of airfoils are emerging as a result of improved understanding of airfoil performance and the ability to design new airfoils using computer methods, the NACA airfoils remain of interest for aerodynamic design. A number of references have been included to allow the reader to study both the older NACA literature and the new airfoil design ideas. Taken together, this literature provides a means of obtaining a rather complete understanding of ways in which airfoils can be shaped to obtain desired performance characteristics.

The NACA airfoils are constructed by combining a thickness envelope with a camber or mean line. The equations which describe this procedure are:

$$X_u = X - Y_t(X) \cdot \sin\theta$$

$$Y_u = Y_c(X) + Y_t(X) \cos\theta$$

and

$$X_L = X + Y_t(X) \sin\theta$$

$$Y_L = Y_c(X) - Y_t(X) \cos\theta$$

where $Y_t(X)$ is the thickness function, $Y_c(X)$ is the camber line function, and

$$\theta = \tan^{-1}\left(\frac{dY_c}{dX}\right)$$

is the camber line slope. It is not unusual to neglect the camber line slope, which simplifies the equations and makes the reverse problem of extracting the thickness envelope and mean line for a given airfoil straightforward.

The next section provides a brief sketch of the development of the NACA airfoils. Although this material is covered in the book by Abbott and von Doenhoff the original reports contain remarkably clear accounts of the manner in which the foils evolved, the methods used in performing the tests, and the analysis of the results. Thus, the original reports should be consulted whenever possible.

EVOLUTION OF THE NACA AIRFOILS	PRIMARY NACA REPORT	AUTHORS	DATE
1. The 4 digit foils: according to Abbott, Pinkerton found that the thickness distribution of the Clark Y and Gottingen 398 foils were similar, and Jacobs selected a function to describe this thickness distribution. The mean lines were selected to be described by two parabolic arcs which were tangent at the position of maximum camber.	R-460	Jacobs, Ward and Pinkerton	1933
2. The 4 digit modified foils: the camber lines were identical to the 4 digit series and a more general thickness distribution was defined, which allowed variations in the leading edge radius and position of maximum thickness to be investigated.	R-492	Stack and von Doenhoff	1934
3. The 5 digit foils: the thickness distribution was kept identical to the 4 digit series and a new camber line was defined which allowed for camber to be concentrated near the leading edge. A reflexed camber line was designed to produce zero pitching moment, but has generally not been used.	R-537 R-610	Jacobs, Pinkerton and Greenberg	1935 1937
4. The 6 series foils: the foils were designed to maintain laminar flow over a large portion of the chord by delaying the adverse pressure gradient. The thickness envelope was designed by using exact airfoil theory and no simple formulas are available to describe the shape. The camber lines were designed using thin airfoil theory and simple formulas are available which describe the shape.	R-824*	Abbott, von Doenhoff and Stivers	1945

* Additional section data is contained in NASA R-84, 1958, by Patterson and Braslow.

EVOLUTION OF THE NACA AIRFOILS	PRIMARY NACA REPORT	AUTHORS	DATE
5. The 6A series foils: the foils were designed to provide sections with simple (nearly straight) surface geometry near the trailing edge, while maintaining the same general properties as the 6 series foils. The camber line can be described by a simple alteration of the standard 6 series mean line.	R-903*	Loftin	1948

ADDITIONAL AIRFOIL REFERENCES

The primary reference volume for all subsonic airfoil studies remains:

Abbott, I. H. and von Doenhoff, A. E., Theory of Wing Sections, Dover, 1949.

Some historical accounts of the NACA airfoil program:

Abbott, I. H., "Airfoils," in Evolution of Aircraft Wing Design, AIAA Symposium, March 1980, AIAA Paper 80-3033.

Jones, R. T., "Recollections From an Earlier Period in American Aeronautics," Ann. Rev. of Fluid Mechanics, Vol. 9, pp. 1-11, 1977.

An extensive and excellent survey of the older airfoils is contained in the English translation of the German book:

Riegels, F. W., Aerofoil Sections, Butterworths, London, 1961.

An entrance to the modern approach to airfoil design can be gained by referring to:

Advanced Technology Airfoil Research, Proceedings of a Conference at NASA Langley, March 1978, Vol. I, Pts. 1 and 2, NASA CP 2045, Vol. II, NASA CP 2046.

Liebeck, R. H., "Design of Airfoils for High Lift," Evolution of Aircraft Wing Design, AIAA Symposium, March 1980, AIAA Paper No. 80-3034.

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* Additional section data is contained in NASA R-84, 1958, by Patterson and Braslow.

NASA has published two reports describing computer programs that produce the NACA airfoil ordinates:

Ladson, C. L. and Brooks, C. W., Jr., "Development of a Computer Program to Obtain Ordinates for NACA 4-Digit, 4-Digit Modified, 5-Digit, and 16-Series Airfoils," NASA TM X-3284, November 1975.

Ladson, C. L. and Brooks, C. W., Jr., "Development of a Computer Program to Obtain Ordinates for NACA 6- and 6A-Series Airfoils," NASA TM X-3069, September 1974. (This program is not accurate for sections less than 6% thick or greater than 15%.)

A number of modern transonic sections have been presented in the series of books by Garabedian and co-workers:

Bauer, F., Garabedian, P. and Korn, D., A Theory of Supercritical Wing Sections with Computer Programs and Examples, Lecture Notes in Economics and Mathematical Systems, Vol. 66, Springer-Verlag, 1972.

Bauer, F., Garabedian, P., Jameson, A. and Korn, D., Supercritical Wing Sections II, A Handbook, Lecture Notes in Economics and Mathematical Systems, Vol. 108, Springer-Verlag, New York, 1975.

Bauer, F., Garabedian, P. and Korn, D., Supercritical Wing Sections III, Lecture Notes in Economics and Mathematical Systems, Vol. 150, Springer-Verlag, New York, 1977.

2.4 THE NACA 4 DIGIT AIRFOIL

This program produces the ordinates of the NACA 4-digit airfoils. If the PC-100C printer is used, a formatted output is generated and if the Math/Utilities Library module is available, a print plot can be generated.

The numbering system for these airfoils is defined by:

NACA MPXX

where XX is the maximum thickness, t/c, in percent chord,

M is the maximum value of the mean line in percent chord,

P is the chordwise position of the maximum camber or mean line, M, in tenths of the chord.

Note that although the numbering system implies integer values, the present program can provide 4 digit foils for arbitrary values of M, P and XX.

An example: NACA 2412 is a 12% thick airfoil, with a max value of the camber line of .02, at x/c = .4.

The thickness distribution is given by:

$$\frac{Y_t}{C} = \left(\frac{t}{c}\right) \cdot \left[a_0 \sqrt{x/c} - a_1 \left(\frac{x}{c}\right) - a_2 \left(\frac{x}{c}\right)^2 + a_3 \left(\frac{x}{c}\right)^3 - a_4 \left(\frac{x}{c}\right)^4 \right]$$

where $a_0 = 1.4845$ $a_2 = 1.7580$ $a_4 = 0.5075$

$a_1 = 0.6300$ $a_3 = 1.4215$

The maximum thickness occurs at x/c = .30, and the leading edge radius is $\left(\frac{r_{LE}}{c}\right) = 1.1019 \left(\frac{t}{c}\right)^2$. The included angle of the trailing edge is:

$$\delta_{TE} = 2 \tan^{-1} \left(1.16925 \cdot \left(\frac{t}{c}\right)\right).$$

The camber line is given by:

$$\left.\begin{aligned}\frac{Y_c}{c} &= \frac{M}{P^2}\left[2P\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right]\\ \frac{dY_c}{dX} &= \frac{2M}{P^2}\left(P - \left(\frac{x}{c}\right)\right)\end{aligned}\right\}\left(\frac{x}{c}\right) < P$$

and

$$\left.\begin{aligned}\frac{Y_c}{c} &= \frac{M}{(1-P)^2}\left[1 - 2P + 2P\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right]\\ \frac{dY_c}{dX} &= \frac{2M}{(1-P)^2}\left[P - \left(\frac{x}{c}\right)\right].\end{aligned}\right\}\left(\frac{x}{c}\right) > P$$

The camberline slope is given by $\theta = \tan^{-1}\left(\frac{dY_c}{dX}\right)$.

The upper and lower ordinates are then computed using the equations given in Section 2.3.

USER INSTRUCTIONS -- PROGRAM 2.4

STEP	ENTER	PRESS	DISPLAY
1. Input max thickness*.	t/c	A	
2. Input max camber*.	M	B	
3. Input position of max* camber (skip if M=0).	P	C	
* All input in actual fractional value.			
4. Compute L.E. radius and trailing edge angle (if desired).	-	SBR π RCL 26	L.E. radius T.E. angle
Without Printer:			
5. Find upper ordinates at x/c	x/c	x≷t A' RCL 54	 Y_u/c X_u/c
6. Find lower ordinates at x/c	-	B' RCL 55	Y_ℓ/c X_ℓ/c
7. To get thickness and camber line (if only Y_t and Y_c are desired, skip step 6).		RCL 38 RCL 39	Y_t/c Y_c/c
(Repeat 5-7 for each x/c.)			
With Printer:			
5P. Input the step size at which ordinates are desired.	Δ X	D	A printer tape is generated from $\frac{x}{c} = 0$ to $\frac{x}{c} = 1$, in Δ X increments.
With Math/Utilities Module:			
6PP. Input number of tape widths used for print plot.	n	E	A print plot of the airfoil section is constructed.

SAMPLE CASE -- NACA 6709

Input: $t/c = .09$ Computed L.E. Radius $\frac{r_{\ell e}}{c} = .0089$

$M = .06$

$P = .70$ T.E. Angle $\delta_{TE} = 12.01°$

x/c	Y_t/c	Y_c/c	X_u/c	Y_u/c	X_ℓ/c	Y_ℓ/c
0.0	.0000	.0000	0.0000	.0000	0.0000	.0000
0.1	.0351	.0159	0.0949	.0507	0.1051	-.0188
0.2	.0430	.0294	0.1948	.0721	0.2052	-.0133
0.4	.0435	.0490	0.3968	.0924	0.4032	.0056
0.6	.0342	.0588	0.5996	.0930	0.6008	.0246
0.8	.0197	.0533	0.8026	.0728	0.7974	.0338
1.0	.0009	.0000	1.0004	.0009	0.9996	-.0009

To set up program: In addition to the basic listing, a number of constants must be stored. They are stored in the third bank (R_{30}-R_{59}) so that only three card sides are needed. Basic airfoil constants are stored in registers R_{31}-R_{36} and R_{59}. Formats for the printer output are stored in R_{42}-R_{53} and R_{57}-R_{58}. In addition to the listing, a copy of the contents of the registers for a sample case is included, allowing the required constants to be copied in directly.

DETAILS OF PROGRAM 2.4

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-128	Primary routine E', given x/c, Y_t/c and Y_c/c are computed, routine $\lvert X \rvert$ is called to compute θ_c.	0	ΔX
		1	X
		2	Number of tapes.
		3	Y_{min}
		4	Y_{max}
129-149	Routine $\lvert X \rvert$ computes θ_c.	5	-
150-183	Routine A', given x/c in t, E' is called and X_u, Y_u are computed and stored.	6	-
		7	-
		8-25	Used by plotter, MU-05.
		26	δ_{TE}
184-215	Routine B', after a call to A', B' is used to compute and store X_ℓ, Y_ℓ at the same x/c as A^1 call.	27	x/c
		28	t/c
		29	M
		30	P
		31	L.E. radius const. = .74226
216-224	Store t/c.	32	a_0 = 1.4845
225-229	Store M.	33	a_1 = 0.6300
		34	a_2 = 1.7580
230-234	Store P.	35	a_3 = 1.4215
235-303	Store ΔX, start printer calculation, print out headings.	36	a_4 = 0.5075
		37	-
304-373	Compute and print out coordinates at each x/c location.	38	Y_t/c
		39	Y_c/c
374-442	Superplotter print plot control.	40	Y_u/c
		41	Y_ℓ/c
443-472	Routine π computes the leading edge radius and trailing edge angle.	42-53	Formats
		54	X_u
		55	X_ℓ
		56	θ_c
		57-58	Formats
		59	T.E. const. = 1.16925

2.5 THE NACA 5 DIGIT AIRFOIL

This program generates the ordinates of the standard NACA 5 digit airfoils. If the PC-100C printer is used, a formatted output is generated. The equations for the reflexed trailing edge case are presented, but not included in the program.

The numbering system for these airfoils is defined by:

NACA LPQXX

where XX is the maximum thickness, t/c , in percent chord.

L is the amount of camber, the design lift coefficient is $\frac{3}{2}$ L, in tenths.

P is the designator for the position of maximum camber, X_f, where X_f = P/2, and P is given in tenths of the chord.

Q = 0; standard 5 digit foil camber
= 1; "reflexed" camber.

An example: NACA 23012, is a 12% thick airfoil, the design lift coefficient is .3, the position of max camber is located at x/c = .15, and the "standard" 5 digit foil camber line is used.

The thickness distribution is the same as the NACA 4 digit airfoil thickness distribution described in 2.4.

The standard camberline is given by:

$$\left.\begin{aligned} \frac{Y_c}{C} &= \frac{K_1}{6}\left[\left(\frac{x}{c}\right)^3 - 3m\left(\frac{x}{c}\right)^2 + m^2(3-m)\left(\frac{x}{c}\right)\right] \\ \frac{dY_c}{dX} &= \frac{K_1}{6}\left[3\left(\frac{x}{c}\right)^2 - 6m\left(\frac{x}{c}\right) + m^2(3-m)\right] \end{aligned}\right\} 0 < \left(\frac{x}{c}\right) < m$$

$$\left.\begin{aligned} Y_c &= \frac{K_1}{6} m^3\left(1-\frac{x}{c}\right) \\ \frac{dY_c}{dX} &= -\frac{K_1}{6} m^3 \end{aligned}\right\} m < \left(\frac{x}{c}\right) < 1$$

m is <u>not</u> the position of max camber, but is related to the max camber position by:

$$X_f = m\left(1 - \sqrt{\frac{m}{3}}\right)$$

K_1 is defined to avoid the leading edge singularity for a prescribed C_{ℓ_i} and m:

$$K_1 = \frac{6\ C_{\ell_i}}{Q}$$

where:

$$Q = \frac{3m - 7m^2 + 8m^3 - 4m^4}{\sqrt{m\ (1-m)}} - \frac{3}{2}\ (1-2m)\ \left[\frac{\pi}{2} - \sin^{-1}(1-2m)\right].$$

Note that K_1 is a linear function of C_{ℓ_i} and the K_1's were originally tabulated for C_{ℓ_i} = .3, and these are multiplied by (C_{ℓ_i}/.3) to get values at other C_{ℓ_i}'s.

Prior to the main calculation, values of Q and K_1 must be determined. In some cases, the computed values of K_1 and Q differ from the official tabulated values. The tabulated values should be used in order to reproduce the official ordinates. The following table illustrates the differences:

MEAN LINE	X_f	m		K_1		
		TABULATED	COMPUTED	TABULATED	USING TAB m	USING COMP m
210	.05	.0580	.0581	361.4	351.56	350.332
220	.10	.1260	.1257	51.65	51.318	51.578
230	.15	.2025	.2027	15.957	15.955	15.920
240	.20	.2900	.2903	6.643	6.641	6.624
250	.25	.3910	.3913	3.230	3.230	3.223

Once the camberline parameters are chosen, the airfoil is constructed using the equations given in Section 2.3.

To obtain the computed values for cases other than those tabulated above, the preliminary calculation program must be used.

The reflexed mean line equations were derived to produce a zero pitching moment about the quarter chord.

$$\frac{Y_c}{c} = \frac{K_1}{6}\left[\left(\frac{x}{c}-m\right)^3 - \frac{K_2}{K_1}(1-m)^3\left(\frac{x}{c}\right) - m^3\left(\frac{x}{c}\right) + m^3\right] \qquad 0 < \left(\frac{x}{c}\right) < m$$

$$= \frac{K_1}{6}\left[\frac{K_2}{K_1}\left(\frac{x}{c}-m\right)^3 - \frac{K_2}{K_1}(1-m)^3\left(\frac{x}{c}\right) - m^3\left(\frac{x}{c}\right) + m^3\right] \qquad m < \left(\frac{x}{c}\right) < 1$$

where

$$\frac{K_2}{K_1} = \frac{3\,(m - X_f)^2 - m^3}{(1-m)^3}$$

The parameters are defined as follows: i) given X_f, find m to given $C_{m_{c/4}} = 0$; ii) given X_f, m, calculate K_1 to given $C_{\ell_i} = .3$.

The tabulated values for these lines are:

MEAN LINE	(P) X_f	M	K_1	K_2/K_1
211	.05	-	-	-
221	.10	.1300	51.99	.000764
231	.15	.2170	15.793	.006770
241	.20	.3180	6.520	.030300
251	.25	.4410	3.191	.135500

USER INSTRUCTIONS -- PROGRAM 2.5a (Preliminary Calculation)

STEP	ENTER	PRESS	DISPLAY
To compute m, use ML-08*:			
1. Input X_f	X_f	STO 10	
2. Start ML-08 (see ML Manual for program description).		2nd Pgm 08	
3. Enter lower limit (∿.0).	$X_{\ell\ell}$	A	
4. Enter upper limit (∿.5).	$X_{u\ell}$	B	
5. Maximum error (∿1. X 10^{-9})	ε	D	
6. Calculate root of equation	-	E	m
To compute K_1:			
1. Input m	m	A	K_1

* ML-08 finds the roots of the equation f(x) = 0 over the interval specified using the graphical bisection method.

USER INSTRUCTIONS -- PROGRAM 2.5 b (Main Airfoil Program

STEP	ENTER	PRESS	DISPLAY
1. Input max thickness (in fraction of chord).	t/c	A	
2. Input camber magnitude const.	K_1	B	
3. Input camber line juncture position.	m	C	
4. If $C_{\ell_i} \neq 3$, input C_{ℓ_i}.	C_{ℓ_i}	E	
5. Compute L.E. radius and T.E. angle (if desired).	-	SBR π RCL 26	L.E. radius T.E. angle
Without Printer:			
6. Find upper ordinates at x/c.	x/c	$X \gtrless t$ A' RCL 54	 Y_u/c X_u/c
7. Find lower ordinates at this x/c.	-	B' RCL 55	Y_ℓ/c X_ℓ/c
8. To get thickness and camber-line (if only Y_t and Y_c are desired, skip step 7).	-	RCL 38 RCL 39	Y_t/c Y_c/c
(Repeat 6-8 for each t/x.)			
With Printer:			
6P. Input the step size at which ordinates are desired.	ΔX	D	A printer tape is generated from $\frac{x}{c}=0$ to $\frac{x}{c}=1$ in ΔX increments.

SAMPLE CASE:

1. For m and K_1, use the results tabulated above to verify the program execution.

2. NACA 23024

Input $t/c = .24$

$K_1 = 15.955$

$m = .2025$

Computed: L.E. Radius $\frac{r_{\ell e}}{c} = .063$

T.E. Angle, $\delta_{TE} = 26.32°$

x/c	Y_t/c	Y_c/c	X_u/c	Y_u/c	X_ℓ/c	Y_ℓ/c
0.0	.0000	.0000	0.0000	.0000	0.0000	0.0000
0.1	.0937	.0170	0.0942	.1105	0.1058	- .0765
0.2	.1148	.0177	0.2025	.1324	0.1975	- .0971
0.4	.1161	.0132	0.4026	.1293	0.3974	- .1028
0.6	.0913	.0088	0.6020	.1001	0.5980	- .0824
0.8	.0525	.0044	0.8012	.0569	0.7988	- .0480
1.0	.0025	.0000	1.0001	.0025	0.9999	- .0025

To set up program: In addition to the basic listing, a number of constants must be stored for the main program. Basic airfoil constants are stored in registers R_{31}-R_{36} and R_{59}. Formats for the printer are stored in R_{23}-R_{24}, R_{42}-R_{53}, and R_{57}-R_{58}. In addition to the listing, a copy of the contents of the registers for a sample case is included, allowing the required constants to be copied in directly. Note that in the printer output, M is designated as X and the Y coordinates are denoted as Z ordinates.

DETAILS OF PROGRAM 2.5

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-159	Primary routine E', given x/c, Y_t/c and Y_c/c are computed, routine $\|X\|$ is called to compute θ_c.	0 2-22 23-24 25	ΔX; 1-X Not used. Printer Format. C_{ℓ_i}
160-168	Routine $\|X\|$, computes θ_c.	26 27	δ_{TE} x/c
169-202	Routine A', given x/c in t, E' is called and X_u, Y_u are computed and stored.	28 29 30 31	t/c K_1 X_1 L.E. radius const.
203-234	Routine B', after a call to A', B' is used to compute and store X_ℓ, Y_ℓ at the same x/c as the A' call.	32 33 34	a_0 a_1 a_2
235-243	Store t/c.	35 36	a_3 a_4
244-252	Store K_1.	37	-
253-257	Store X_1.	38 39	Y_t/c Y_c/c
258-357	Store ΔX, start printer calculation print out headings.	40 41	Y_u/c Y_ℓ/c
358-426	Compute and print out coordinates at each x/c location.	42-53 54	Formats X_u
427-456	Routine π, computes the leading edge radius and trailing edge angle.	55 56 57-58	X_ℓ θ_c Formats
457-472	Store C_{ℓ_i} and redefine K_1.	59	T.E. const. = 1.16925

2.6 THE NACA MODIFIED 4 DIGIT AIRFOIL

This program generates the ordinates of the NACA modified 4 digit airfoils. If the PC-100C printer is used, a formatted output is generated. The NACA 16 series airfoils are a subset of this series.

The numbering system for these airfoils is defined by:

NACA MPXX-IT

where MPXX is the standard 4 digit designation and the IT appended at the end describes the modification to the thickness distribution. These are defined as:

T -- chordwise position of maximum thickness;

I -- designation of the leading edge radius;

$$\frac{r_{\ell e}}{c} = 1.1019 \left(\frac{I}{6} \cdot \frac{t}{c}\right)^2$$

for $I \leq 8$

and $$\frac{r_{\ell e}}{c} = 3 \times 1.1019 \left(\frac{t}{c}\right)^2 \quad \text{for } I = 9.$$

I = 6 produces the leading edge radius of the standard 4 digit airfoils.

An example: NACA 0012-74 denotes an uncambered 12% thick airfoil, with a maximum thickness at x/c = .40 and a leading edge radius of .0216, which is 36% larger than the standard 4 digit value.

The NACA 16 series is a special case of the modified 4 digit airfoil with a leading edge index of I = 4 and the maximum thickness located at x/c = .5 (T = 5). As an example, the NACA 16-012 is equivalent to an NACA 0012-45.

The thickness distribution is given by:

$$\frac{Y_t}{c} = 5\left(\frac{t}{c}\right)\left[a_0\sqrt{(x/c)} + a_1\left(\frac{x}{c}\right) + a_2\left(\frac{x}{c}\right)^2 + a_3\left(\frac{x}{c}\right)^3\right] \qquad 0 < \frac{x}{c} < T$$

and

$$\frac{Y_t}{c} = 5\left(\frac{t}{c}\right)\left[.002 + d_1(1-X) + d_2(1-X)^2 + d_3(1-X)^3\right] \qquad T < \frac{x}{c} < 1$$

The coefficients are determined by solving for the d's first, based on the trailing edge slope and the condition of maximum thickness at x/c = T. Once these constants are found, the a's are found by relating a_0 to the specified leading edge radius, the maximum thickness at x/c = T, and continuity of the curvature at x/c = T. These constants are all determined for t/c = .20, and then scaled to the other values by multiplying by the factor 5·(t/c). The value of d_1 controls the trailing edge slope and was originally selected to avoid reversals of curvature. In addition to the tabulated values, Riegels has provided an interpolation formula.

The tabulated and approximate values of d_1 are given in the following table:

T	TAB. d_1	APPROX. d_1
.2	.200	.200
.3	.234	.234
.4	.315	.314
.5	.465	.464
.6	.700	.722

where the Riegels approximation is given by:

$$d_1 \cong \frac{(2.24 - 5.42T + 12.3T^2)}{10\,(1 - .878T)} \quad .$$

Once the value of d_1 is known, d_2 and d_3 can be found from the relations given by Riegels:

$$d_2 = \frac{.294 - 2\,(1-T)\,d_1}{(1-T)^2}$$

and

$$d_3 = \frac{-.196 + (1-T)\,d_1}{(1-T)^3}$$

With the d's determined, the a's can be found. a_0 is based on the leading edge radius:

$$a_0 = .296904 \cdot \chi_{LE}$$

where $\chi_{LE} = \frac{I}{6}$ for $I \leq 8$

$= 10.3923$ for I = 9.

Defining:

$$\rho_1 = \left(\frac{1}{5}\right) \frac{(1-T)^2}{[.588 - 2d_1(1-T)]} \quad ;$$

$$a_1 = \frac{.3}{T} - \frac{15}{8} \cdot \frac{a_0}{\sqrt{T}} - \frac{T}{10\rho_1}$$

$$a_2 = -\frac{.3}{T^2} + \frac{5}{4} \cdot \frac{a_0}{T^{3/2}} + \frac{1}{5\rho_1}$$

$$a_3 = \frac{.1}{T^3} - \frac{.375\ a_0}{T^{5/2}} - \frac{1}{10\rho_1 T} \quad .$$

The camber lines are identical to the standard 4 digit airfoils described in Section 2.4. The upper and lower ordinates are then computed using equations given in Section 2.3.

To obtain the values of the coefficients for the main calculation, a preliminary program must be executed.

USER INSTRUCTIONS -- PROGRAM 2.6a

STEP	ENTER	PRESS	DISPLAY
1. Enter the leading edge radius constant.	χ_{LE}	A	
2. To determine d_1 from Riegels approx.	T	B	d_1
or			
2A. To input tabulated or other values of d_1	d_1	C	
3. Input point of max thickness and compute coefficients.	T	D	-
		RCL 10 RCL 11 RCL 12 RCL 13	a_0 a_1 a_2 a_3
		RCL 15 RCL 16 RCL 17 RCL 18	d_0 d_1 d_2 d_3

USER INSTRUCTIONS -- PROGRAM 2.6

STEP	ENTER	PRESS	DISPLAY
1. Store baseline coefficients	a_0 a_1 a_2 a_3 d_0 d_1 d_2 d_3	STO 32 33 34 35 19 20 ↓ 21 STO 22	
2. Store leading edge radius constant (echoed only - not used).	χ_{LE}	STO 16	
3. Input maximum thickness (fractional value).	t/c	A	
4. Input maximum thickness chordwise position.	T	B	
5. Input maximum camber.	M	C	
6. Input position of max camber (required only if M ≠ 0).	P	D	
7. Compute L.E. radius and trailing edge angle (if desired).		SBR π RCL 26	L.E. Radius T.E. Angle
Without Printer:			
8. Find upper ordinates at x/c	x/c	$x \gtrless t$ A' RCL 54	 Y_u/c X_u/c
9. Find lower ordinates at x/c	-	B' RCL 55	Y_ℓ/c X_ℓ/c
10. To get camber and thickness (if only Y_t and Y_c are desired, skip step 9).	-	RCL 38 RCL 39	Y_t/c Y_c/c
(Repeat 8-10 for each x/c.)			
With Printer:			
8P. Input the step size at which ordinates are desired.	ΔX	E	A printer tape is generated from X=0 to X=1 in ΔX increments.

SAMPLE CASE: NACA 0009-74

1. Program 2.6A

Input: χ_{LE} = 7/6 = 1.1667

Approx. d_1

T = .4

Compute:

a_0 = .3464
a_1 = .3940
a_2 = .4218
a_3 = .4527

d_0 = .0020
d_1 = .3144
d_2 =-.2314
d_3 =-.0340

2. Program 2.6

Input: a, d and χ_{LE}

t/c = .09

T = .40

M = 0

Computed L.E. Radius $\frac{r_{\ell e}}{c}$ = .0121

T.E. Angle δ_{TE} = 16.11°

x/c	Y_t/c	Y_c/c
0.0	0.0000	0.0
0.1	0.0333	↓
0.2	0.0402	↓
0.4	0.0450	↓
0.6	0.0399	↓
0.8	0.0249	↓
1.0	0.0009	0.0

To set up the program, the constants of a typical run are included along with the program listing. No stored constants are required, but if the printer is used, the values in R_{14}-R_{15}, R_{42}-R_{53} and R_{57}-R_{58} should be stored to provide the output format.

DETAILS OF PROGRAM 2.6

STEPS	PROGRAM DESCRIPTION
0-175	Primary routine E', given x/c, Y_t/c and Y_c/c are computed, routine |X| is called to compute θ_c.
176-196	Routine |X|, computes θ_c.
197-230	Routine A', given x/c in t, E' is called and X_u, Y_u are computed and stored.
231-262	Routine B', after a call to A', B' is used to compute and store X_ℓ, Y_ℓ at the same x/c as the A' call.
263-271	Store t/c.
272-276	Store M.
277-281	Store P.
282-286	Store T.
287-371	Store ΔX, start printer calculations and print out headings.
372-441	Compute and print out coordinates at each x/c location.
442-475	Routine π, computes the leading edge radius and trailing edge angle.

R	REGISTER CONTENTS
0	ΔX
1	X
2-13	Not used.
14-15	Formats
16	X_{LE}
17	1-X
18	T
19	d_0
20	d_1
21	d_2
22	d_3
23	-
24	-
25	-
26	δ_{TE}
27	x/c
28	t/c
29	M
30	P
31	-
32	a_0
33	a_1
34	a_2
35	a_3
36	-
37	-
38	Y_t/c
39	Y_c/c
40	Y_u/c
41	Y_ℓ/c
42-53	Formats
54	X_u/c
55	X_ℓ/c
56	θ_c
57-58	Formats
59	-

2.7 6 AND 6A SERIES MEAN LINES

The 6 series mean lines were designed using thin airfoil theory to produce a constant loading from the leading edge back to x/c = a, after which the loading decreases linearly to zero at the trailing edge. Theoretically, the loading at the leading edge must be either zero or infinite within the context of thin airfoil theory analysis. The violation of the theory by the assumed finite leading edge loading is reflected by an infinite slope of the camberline at the leading edge. Therefore, according to Abbott and von Doenhoff, the 6 series airfoils were constructed by holding the slope of the mean line constant in front of x/c = .005, with the value at that point. Recall that for round leading edges, the camberline values are essentially not used at points ahead of the origin of the leading edge radius. The theory is discussed by Abbott and von Doenhoff on pages 73-75, 113 and 120. Tabulated values are contained on pages 394-405.

By simply adding various mean lines together, other load distributions can be constructed.

The 6A series airfoils employed an empirical modification of the a = .8 camberline to allow the airfoil to be constructed of nearly straight segments near the trailing edge. This camberline is described by Loftin in NACA R-903.

BASIC CAMBERLINE EQUATIONS:

When a = 1 (a uniform load along the entire chord):

$$\frac{y}{c} = -\frac{c_{\ell_i}}{4\pi} \cdot \left[\left(1-\frac{x}{c}\right) \ln\left(1-\frac{x}{c}\right) + \frac{x}{c} \ln \frac{x}{c} \right]$$

and

$$\frac{dy}{dx} = \frac{c_{\ell_i}}{4\pi} \left[\ln\left(1-\frac{x}{c}\right) - \ln \frac{x}{c} \right]$$

where c_{ℓ_i} is the "ideal" or design lift coefficient, which occurs at zero angle-of-attack.

For a < 1,

$$\frac{y}{c} = \frac{c_{\ell_i}}{2\pi\,(1+a)} \left\{ \frac{1}{1-a} \left[\frac{1}{2}\left(a-\frac{x}{c}\right)^2 \ln \left|a-\frac{x}{c}\right| - \frac{1}{2}\left(1-\frac{x}{c}\right)^2 \ln\left(1-\frac{x}{c}\right) + \frac{1}{4}\left(1-\frac{x}{c}\right)^2 - \frac{1}{4}\left(a-\frac{x}{c}\right)^2 \right] - \frac{x}{c} \ln \frac{x}{c} + g - h \cdot \frac{x}{c} \right\}$$

With $g = \frac{-1}{1-a}\left[a^2 \cdot \left(\frac{1}{2}\ln a - \frac{1}{4}\right) + \frac{1}{4}\right]$

$$h = (1-a)\left[\frac{1}{2}\ln(1-a) - \frac{1}{4}\right] + g$$

and

$$\frac{dy}{dx} = \frac{C_{\ell_i}}{2\pi(1+a)} \cdot \left\{\frac{1}{1-a}\left[\left(1-\frac{x}{c}\right)\ln\left(1-\frac{x}{c}\right) - \left(a-\frac{x}{c}\right)\ln\left(a-\frac{x}{c}\right)\right] - \ln\frac{x}{c} - 1 - h\right\}$$

The associated angle-of-attack is:

$$\alpha_i = \frac{C_{\ell_i} \cdot h}{2\pi(1+a)}$$

<u>a = .8 (Modified)</u>, (the 6A series mean line)

For $0 < \frac{x}{c} < .87437$, use the basic a = .8 camberline, but with a modified value of $C_{\ell_{i_{MOD}}}$ equal to $C_{\ell_i}/1.0209$. For $.87437 < \frac{x}{c} < 1$, use the linear equation:

$$\frac{Y_c}{C_{\ell_i}} = .0302164 - .245209\,(X - .87437)$$

and

$$\frac{dy}{dx} = -.245209 \cdot C_{\ell_i}.$$

Note that at X = 1, the foregoing approximate relation gives Y = -.000589, indicating an α shift of .034° for $C_{\ell_i} = 1$.

USER INSTRUCTIONS -- PROGRAM 2.7

STEP	ENTER	PRESS	DISPLAY
1. If $C_{\ell_i} = 1$	-	A	
or			
if $C_{\ell_i} \neq 1$	C_{ℓ_i}	B	
2. If a = 1	x/c	C RCL 09	y/c dy/dx
Repeat 2. for each x/c desired.			
3. If a < 1	a	D	
4. Enter chord station	x/c	E RCL 09	y/c dy/dx
Repeat 4. for each x/c desired.			

STEPS	PROGRAM DESCRIPTION	R	REGISTER CONTENTS
0-16	Establish C_{ℓ_i}.	0	C_{ℓ_i}
		1	x/c
17-57	For a = 1, find y/c.	2	1-x/c
		3	y/c
58-85	For a = 1, find dy/dx.	4	a
		5	1-a
86-153	For a < 1, compute g and h.	6	g
		7	h
153-252	For a < 1, find y/c.	8	a-x/c
		9	dy/dx
253-309	For a < 1, find dy/dx.		

SAMPLE CASES: (C_{ℓ_i} = 1.0)

	a = .6		a = .8		a = 1.0		MODIFIED .8 (A SERIES)		
x/c	y/c	d y/c/x/c	y/c	d y/c/dx/c	y/c	d y/c/dx/c	x/c	y/c	d y/c/dx/c
.005	.00324	.548243	.00287	.485353	.00251	.421228	.005	.00281	.475417
.050	.02080	.313269	.01841	.277178	.01580	.234311	.050	.01803	.271504
.100	.03437	.237275	.03043	.210504	.02587	.174850	.100	.02980	.206194
.200	.05349	.152473	.04748	.137338	.03982	.110318	.200	.04651	.134526
.400	.07233	.040600	.06528	.046005	.05356	.032266	.400	.06395	.045064
.600	.06880	-.094705	.06644	-.036111	.05356	-.032266	.600	.06508	-.035372
.800	.03691	-.192270	.04771	-.184123	.03982	-.110318	.800	.04674	-.180353
1.000	.00000	-.145518	.00000	-.20385	.00000	$-\infty$	1.000	-.00059	-.245209

NOTE: When x/c = 1. and x/c = a, the display may flash, but the results are correct.

APPENDIX A. DESCRIPTION OF SYMBOLS USED IN THE PROGRAM LISTINGS

The symbols used in the program listings were generated by the PC-100C printer and the symbols are described on page VI-6 of "Personal Programming," the TI58/59 owner's manual. However, a brief description of the symbols is presented to allow users of other types of calculators to convert the programs. Most instruction sets are not too different than those used here, so it should not be difficult to convert the programs given the following information. In this description, the current value in the calculator is referred to as the display register and there is a register used for testing values in order to make transfers called the T register. The types of symbols are broken up into logical categories.

Class 1: Numerical Operations

(,), -, +, X, ÷, =	The standard algebraic notation.
.	Decimal point.
π	The constant 3.141. . .
+/-	An operator changing the sign of the value in the display register. When occurring after EE or exponent entry, it changes the sign of the exponent.
COS*, SIN*, TAN*	Standard trigonometric operations on contents of display register.
LNX*, LOG*	Natural and common logarithm, respectively. Operates on contents of display register.
X^2, $\|X\|$, 1/X, $\sqrt{X}$	Perform indicated operation on contents of display register.
Y^{X}*	Raise value to power specified by following number.
X $\gtrless$ T	Exchanges value of display register with the value of T register.
CE	When occurring in a program, it is called the dummy operation. CE reenters the display value again without having to key the value again.
EE*	Scientific notation, the number following EE is an exponent of 10.
FIX*	Set the number of digits following the decimal point in the display.
INT*	Takes only the integer value of the display register.

* When preceded by INV, the inverse operation occurs. See Appendix B for more details.

Class 2: Register Operations

These symbols precede the register upon which they operate.

* Denotes an indirect operation, wherein, the program looks in the register designated to find the number of the register upon which the actual operation takes place.

STO ST*	Store value in register.
RCL RC*	Recall value in register.
SUM SM*	Add display register value to contents of register.
PRD PD*	Multiply value of display register with value in register and store product in register.
EXC EX*	Exchange value of display register with value in designated register.
CP	In program listing, the value in the T register is set to zero.
CMS	All data memories are set to zero.
OP20-29 OP30-39 OP*	Increment a data register 0-9 by 1. Decrement a data register 0-9 by 1.

Class 3: Transfer Instructions

RST	Positions pointer at instruction 000.
SBR*	Subroutine call, program transfers to label designated in next step after SBR. (Program will return to this location after subroutine.)
GTO GT*	Program will transfer to label designated in next step after GTO.
DSZ*	Decrement and skip on zero. DSZ X N where X is a register 0-9 and when the contents in X are not equal to zero, program transfers to label N.
EQ*	If $X = t$ } the program transfer to the label that is specified by the next instruction.
GE*	If $X \geq t$ } the program transfer to the label that is specified by the next instruction.
STF*	Set a flag (0-9).
IFF*	For IFF y N, if flag y is set, the program transfers to N.

* When preceded by INV, the inverse operation occurs. See Appendix B for more details.

Class 4: Labels

LBL	Designates the following instruction to be a label. This designates the position for transfers from other places in the program.
A - E A' - E'	User defined labels. Normally used for program operation. SBR is not needed when one of these instructions is encountered in the program.

Class 5: Other Instructions

PGM	Refers to built in program modules; these programs are rarely used in AEROCAL. Mainly used for printer routines or register initializing.
R/S	Reverses the status of processing. Either starts or stops program, depending on whether the program was previously running or stopped.
NOP	A null instruction. This instruction has no effect on the program.

Class 6: Printer

ADV	Advances printer one space.
PRT	Prints present value.

Class 7: Mode

DEG	Sets calculator to operate with degrees.
RAD	Sets calculator to operate with radians.

NOTE: INV RAD ≠ DEG, beware.

APPENDIX B. SOME NOTES ON TI59 USE

1. Card Storage

The calculator stores its program and memory on two cards with two banks on each card, with the following divisions:

CARD BANK	PROGRAM STEPS	DATA STORAGE
1	0 - 239	90 - 99
2	240 - 479	60 - 89
3	480 - 719	30 - 59
4	720 - 959	0 - 29

2. Partitioning

The division between program steps and memory can be made in the following increments:

To repartition, enter the number of 10's of memory registers and press 2nd OP 17. The new split is then shown in the display.

To check partitioning, press 2nd OP 16. The split between steps and memory is then shown in the display.

STEPS	MEMORY
959	00
879	09
799	19
719	29
639	39
559	49
479	59 (Default)
399	69
319	79
239	89
159	99

NOTE: 00 counts here so that 09 implies 10 registers and 799 implies 800 program steps.

3. Indirect Function

When used after one of the following operations, the data register is examined and the operation is carried out on the register number stored in the specified register.

Allowable Functions: STO, RCL, EXC, SUM, PRD, FIX
(Typical in AEROCAL)

$X = t$, $X \geq t$, ST FLG, IF FLG, DSZ

Example:

i) The value contained in register 23 is 10.

ii) Consider 5 / STO IND / 23 or in the listing: 5 / ST* / 23

iii) In this case, the value 5 is stored in register 10.

4. Inverse Function

When INV is used before some functions, the inverse operation is carried out. AEROCAL listings use this function for the following operations:

FUNCTION	INVERSE FUNCTION	FUNCTION	INVERSE
Log X	10^x	If FLG	If no flag
ℓn X	e^x	ST FLG	Reset flag
y^x	$\sqrt[x]{y}$	DSZ	Skip on non-zero
Sin	Sin^{-1}	PROD	Divide into memory
Cos	Cos^{-1}	SUM	Subtract from memory
Tan	Tan^{-1}	EE	Removes EE
$X = t$	$X \neq t$	INT	Fractional part
$X \geq t$	$X < t$	SBR	Return

5. OP Functions

These are special control operations. The following are of interest to AEROCAL users:

Code nn	
00 - 07	For printer.
08	List the labels used in the program and their location.
11 - 15	Statistics.
16 - 17	See 2., Partitioning.
20 - 29	Increment a data register 0 - 9 by 1.
30 - 39	Decrement a data register 0 - 9 by 1.

6. Printer Use

To list program, press RST to start at front, or GTO XYZ to start at the step XYZ and press second LIST.

To list data registers, enter 0 and press INV 2nd LIST.

To list labels in program, press RST and 2nd OP 08.

AEROCAL