



AIAA 95-0741

**On the Use of the Potential Flow Model
for Aerodynamic Design at
Transonic Speeds**

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**33rd Aerospace Sciences
Meeting & Exhibit**

January 9-12, 1995 / Reno, NV

On the Use of the Potential Flow Model for Aerodynamic Design at Transonic Speeds

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Abstract

The full potential flow model is still widely used. In this paper we provide a new explanation of its principle shortcoming in terms that can be easily understood by aerodynamicists. The classical nozzle problem is examined to show that the use of the full potential shock jump precludes the influence of back pressure in locating the jump in the nozzle. Although the difference in properties between the potential and Rankine-Hugoniot jumps do not look to be that different when viewed in the customary manner, an analysis by Laitone some years ago reveals startling differences between the potential and Rankine-Hugoniot jumps. This difference provides an improved understanding of the failure of the full potential theory as soon as even modest shock strengths occur. It also suggests a physical basis for the non-uniqueness of potential solutions.

Introduction

The potential flow approximation was the dominate model for transonic aerodynamics for many years. It is still widely used in engineering applications,¹ and can be used effectively in aerodynamic design and analysis. Indeed, considerable effort is still being devoted to the development of potential flow methods for use in design, *e.g.*, Huffman, *et al.*² The potential methods may even be of interest for use in multidisciplinary design methodology³ because of the comparatively small computational cost.

The properties of potential flow jumps were examined by several investigators when numerical solutions of the potential flow equation started being computed for transonic flows including shocks. It was found in the 1980s, when reliable Euler solutions became routinely available for transonic flow calculations, that the full potential solutions were seriously in error when shock waves became even moderately strong ($M_S > 1.25$). Both shock strength and position were incorrect. That there was an erroneous prediction was not surprising since the jump relations for the potential flow equation are physically incorrect. However, the magnitude of the error was surprising. Figure 1 shows

an example. In Fig. 1(a), pressure distributions are compared for Euler and full potential equation solutions over an NACA 0012 airfoil at $M = 0.8$, $\alpha = 0^\circ$. There is only a small difference between the results. The full potential solution has a slightly stronger shock that is just slightly aft of the Euler result. Increasing the angle of attack to 1.25° leads to a stronger shock on the upper surface, which moves aft 13% of the chord. The effect of increasing the angle of attack on the Euler solution is shown in Fig. 1(b). Next we compare the full potential solution with the Euler solution for the 1.25° angle of attack case. Now the difference between the potential and Euler results, given in Fig. 1(c), is extremely large. In this case the potential equation solution shock has moved to the trailing edge, a shock movement of 50% chord. Although this is an extreme case, it is indicative of the large differences between Euler and full potential results when the shocks are strong.**

This paper provides an explanation of the problems with the full potential solutions in physical terms. The differences between the correct and potential jumps are illustrated in a manner that may also explain the non-unique solution problems.

Shock Jump Wave Drag Studies

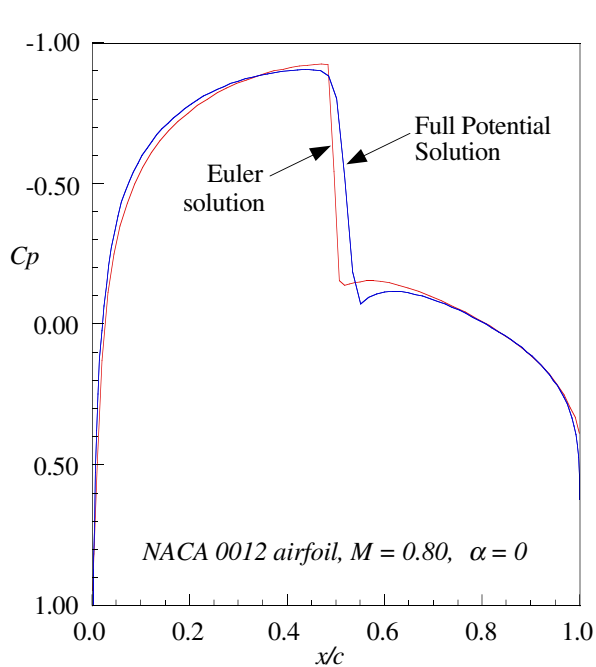
One of the first analyses of the differences between the Rankine-Hugoniot and potential flow (isentropic) jump characteristics was presented by Steger and Baldwin.^{6,7} Further analysis of the isentropic jump relations was carried out by Van der Vooren and Slooff.⁸ Murman and Cole also examined this problem.⁹ Under the full potential assumption, entropy was held constant across the discontinuity, and mass was conserved while momentum was not. This is the traditional full potential formulation. These studies were primarily concerned with the possibility of using the isentropic equation to predict drag. The basic accuracy of the method was not yet in question.

Subsequently, Klopfer and Nixon¹⁰ reexamined the basis for potential flow when shocks were present and proposed a nonisentropic potential formulation. Examining possible

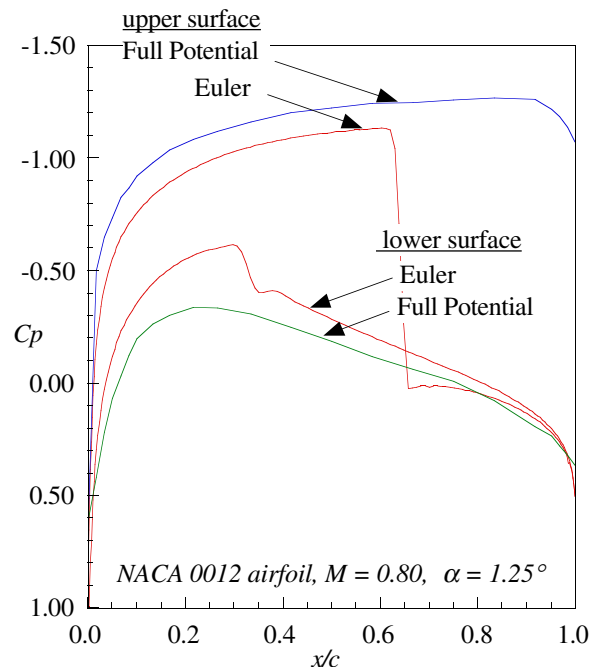
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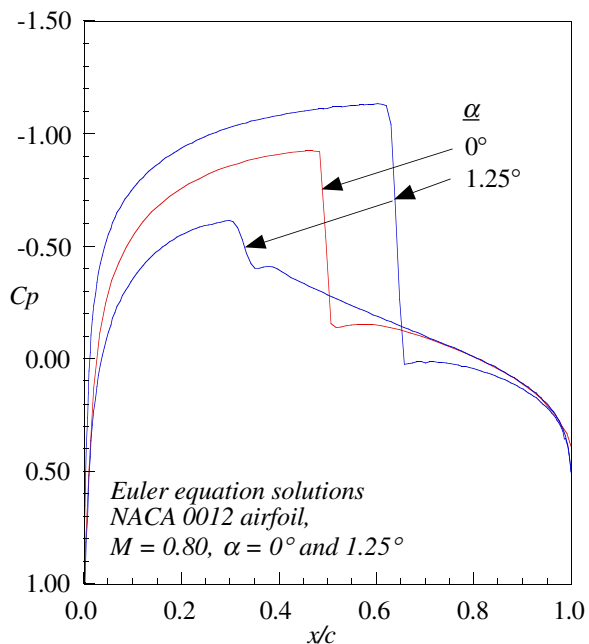
** The Euler equation solutions were provided by Robert Narducci of Virginia Tech, the full potential solutions were from FLO36⁴ ($\alpha = 0^\circ$) and TAIR⁵ ($\alpha = 1.25^\circ$).



a. comparison between full potential and Euler solution for a “weak” shock case.



c. comparison between full potential and Euler solution for a “strong” shock case.



b. effect of change of angle of attack on pressure distribution predicted by the Euler equations

Figure 1. Illustration of the agreement between Euler and full potential solutions at different conditions

jump formulations, they showed that a potential jump which conserved momentum instead of energy was much closer to the actual Rankine-Hugoniot jump condition. They then presented a method using the momentum con-

Figure 1. Illustration of the agreement between Euler and full potential solutions at different conditions (concluded).

servation condition to obtain solutions almost identical to the Euler results for the cases shown above. Lucchi¹¹ also presented a method of using the potential equation with a modified jump condition to obtain results that agreed with the Euler solutions for these same examples. Both Klopfer and Nixon¹⁰ and Lucchi¹¹ also make changes to the Kutta condition. Hafez and Lovell have also investigated this problem and proposed a correction method.¹²

At supersonic speeds Siclari and Visich and Siclari¹³ and Rubel¹⁴ demonstrated that by correcting the isentropic jump at the bow shock, the potential flow equation could be used to obtain accurate results for extremely strong shocks, with freestream Mach numbers as high as ten. Their fundamental analysis was based on the conical flow over circular cones at zero angle of attack, and they presented results for a variety of jump conditions in a manner analogous to the study by Klopfer and Nixon.¹⁰

Nonunique Solutions

Another problem with the solution of the full potential equation was the discovery that the solution was nonunique by Steinhoff and Jameson.¹⁵ It appeared that the incorrect shock jump using the potential flow model leads directly to the nonuniqueness problem.^{15,16} Further discussion is contained in Hirsch’s textbook.¹⁷ However, a more recent analysis by McGratten¹⁸ produced nonunique solutions for the Euler equations also. In this case the shock was fairly weak, and he concluded that: “it is unlikely that it is due to

the isentropic assumption of the potential model". The implication is that the details of the treatment of the flow at the trailing edge through the Kutta condition or its equivalent for the Euler equation is equally important in producing nonunique solutions.

Classical Isentropic Jump

Although the breakdown in the isentropic flow model is expected, the suddenness with which the breakdown occurs is surprising. Numerous computational results have demonstrated this very sudden breakdown in the potential flow approximation. The potential flow equation solution agrees with Euler equation results for cases with weak shock waves. The agreement then seems to deteriorate quite suddenly as the shock becomes stronger. To quantize the difference between the actual and potential flow models, it is common to compare the shock jump Mach number between the classical isentropic jump and the Rankine-Hugoniot. Unfortunately, there is no closed form solution in the case of the isentropic jump, and a numerical solution is required. An especially attractive approach to the solution is given in Appendix C of the report by Steger and Baldwin.⁶ Defining

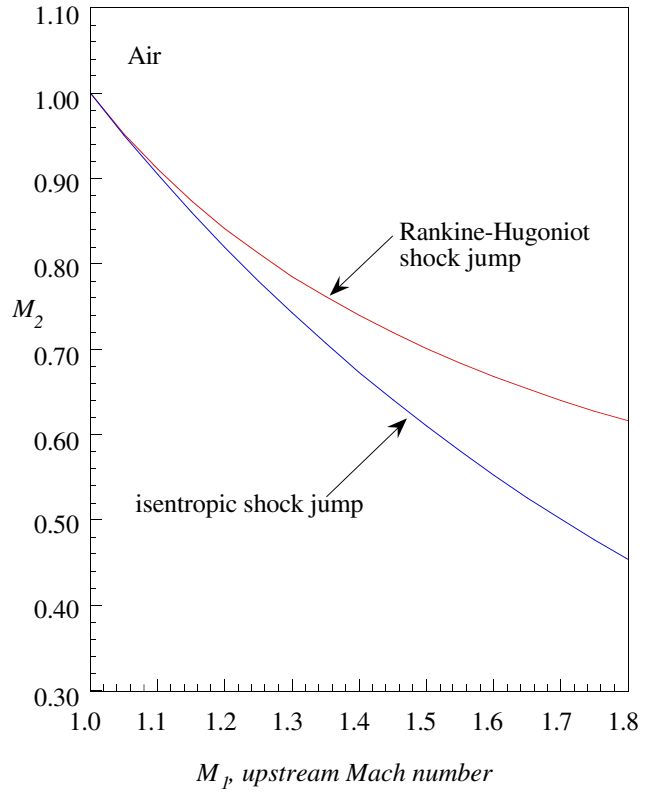
$$r = \left(\frac{M_2^2}{M_1^2} \right)^{\frac{\gamma-1}{\gamma+1}} \quad (1)$$

the value of M_2 can be found (after use of some identities), by solving the polynomial

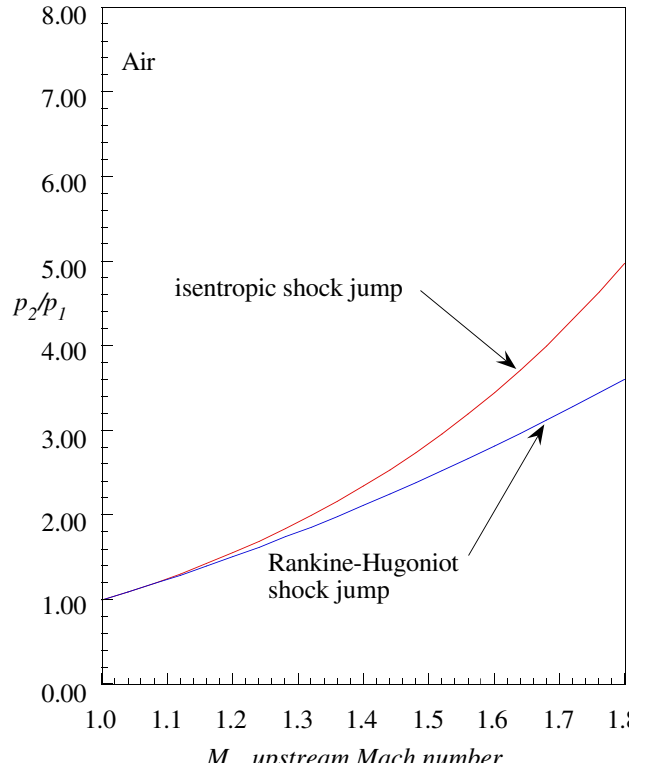
$$r + r^2 + r^3 + r^4 + r^5 = \frac{2}{(\gamma-1)M_1^2} \quad (2)$$

The solution can be found by Newton's method using about three lines of code.

The result from Eqn. 2 is shown compared with the Rankine-Hugoniot jump in Fig. 2(a). In this figure the potential flow shock jump is seen to be asymptotically correct as the upstream Mach number goes to unity, and to depart smoothly from the real jump condition as the upstream Mach number increases. The potential flow shock jump is larger than the Rankine-Hugoniot jump. This figure does not suggest a sudden breakdown. The error between the isentropic and Rankine-Hugoniot jump values is not too different between $M = 1.2$ and $M = 1.3$. Yet the expectation, as illustrated in Fig. 1, would be that the difference between Euler and potential solutions would be much greater for the $M = 1.3$ case. The other typical comparison between the Rankine-Hugoniot and isentropic jump conditions is the pressure ratio, p_2/p_1 . This comparison is given in Fig. 2(b). The difference between the two cases grows smoothly, and is consistent with the Mach number differences in Fig. 2(a).



a. comparison of Mach jumps



b. comparison of pressure jump

Figure 2. Comparison of classical isentropic jump with the Rankine-Hugoniot condition values.

Laitone's Analysis

Recent evaluations of the aerodynamic methods used in ACSYNT¹⁹ revealed the importance of early work by Laitone.²⁰ ACSYNT²¹ is an aircraft sizing and optimization program. Many of the key aerodynamic estimates in ACSYNT are based on approximations developed by John Axelson,²² and implemented in AEROX.²³ A variety of approximations are used to obtain aerodynamic characteristics. One of these approximations is based on the transonic flow limiting velocity obtained from momentum relations by Laitone.²⁰ This work predates the modern transonic era and has been overlooked, except by Axelson.

Laitone derived a condition for a limiting velocity expected to occur over an airfoil at transonic speeds. It is the ratio of static pressure behind the shock to the stagnation pressure ahead of the shock.* Laitone only considered the Rankine-Hugoniot jump conditions. However, when this condition is examined for both the Rankine-Hugoniot and isentropic shock relations, a remarkable difference occurs, as shown in Fig. 3. Using the Rankine-Hugoniot conditions, the maximum Mach number corresponding to this pressure is found to be $M = [(\gamma + 3)/2]^{1/2}$, or 1.483 for air. In contrast, no limit occurs for the isentropic jump relations. As the shock Mach number approaches 1.3, a dramatic difference between the Rankine-Hugoniot and isentropic model develops. Laitone gave the following explanation of the significance of the Rankine-Hugoniot result:

“If $M_1 > 1.483$ then p_2 is less than it would be for $M_1 = 1.483$. Therefore the downstream pressure, which is always greater than p_2 , can force the normal shock upstream until the location where $M_1 = 1.483$ is reached, since the entire flowfield behind the shock is subsonic.”

Laitone is defining what amounts to a stability criteria. This natural limiting phenomena is completely absent in the isentropic shock jump model, which does not have a maximum value or slope reversal with Mach number. He is also pointing out the importance of the downstream pressure field in controlling shock position and strength. A subsequent paper by Laitone provides further justification for this limit.²⁴ Note that inviscid conditions are of interest here, and viscous effects would prevent this limit from occurring in an actual flowfield. However, aerodynamic designers and optimization methods often use inviscid models for initial design work. The potential model continues to offer advantages in terms of storage and execution speed. With the widespread use of workstations, potential solutions continue to be used. The users of these methods need to be aware of the results presented in Fig. 3.

* This relation is also given in NACA 1135, although it is not tabulated, and the characteristic property of the relation was not discussed.

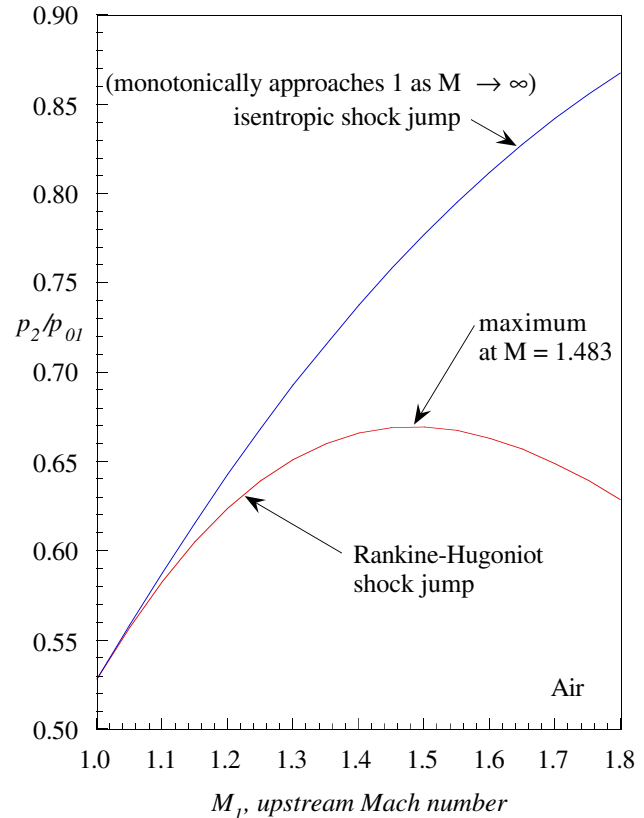


Figure 3. Laitone's ratio, the ratio of the static pressure behind a normal shock to the stagnation pressure in front of the shock.

The 1D Nozzle Problem Example**

The importance of the downstream flowfield pressure on shock position can also be illustrated by considering the case of quasi-one dimensional flow in a nozzle. Consider first the usual (Rankine-Hugoniot jump) case. Over a range of specified exit pressures, the exit pressure controls the position of the shock wave in the nozzle. The equivalent situation fails to occur using the potential flow model. As shown in Fig. 4, the potential flow model jump is simply a jump from the supercritical curve to the subcritical curve. The isentropic jump can satisfy only one distinct value of the exit pressure. Hirsch¹⁷ also used this property to illustrate the problem of solution nonuniqueness using the potential flow model. The one specific subcritical value of exit pressure can be obtained with a shock located anywhere in the divergent portion of the nozzle. A more physical and troubling interpretation is possible. The nozzle problem shows that the potential flow model cannot respond to a change in exit, or downstream, pressure by adjusting the shock location. This accounts for the erroneous shock position predictions in potential flow calculations once the shock strength becomes strong, and has not been identified explicitly previously. It also shows why the conditions at the airfoil trailing edge are so important.

** During the final preparation of the paper, I discovered that Hirsch¹⁷ has already used this problem for the same purpose in his Volume I, pages 122-123.

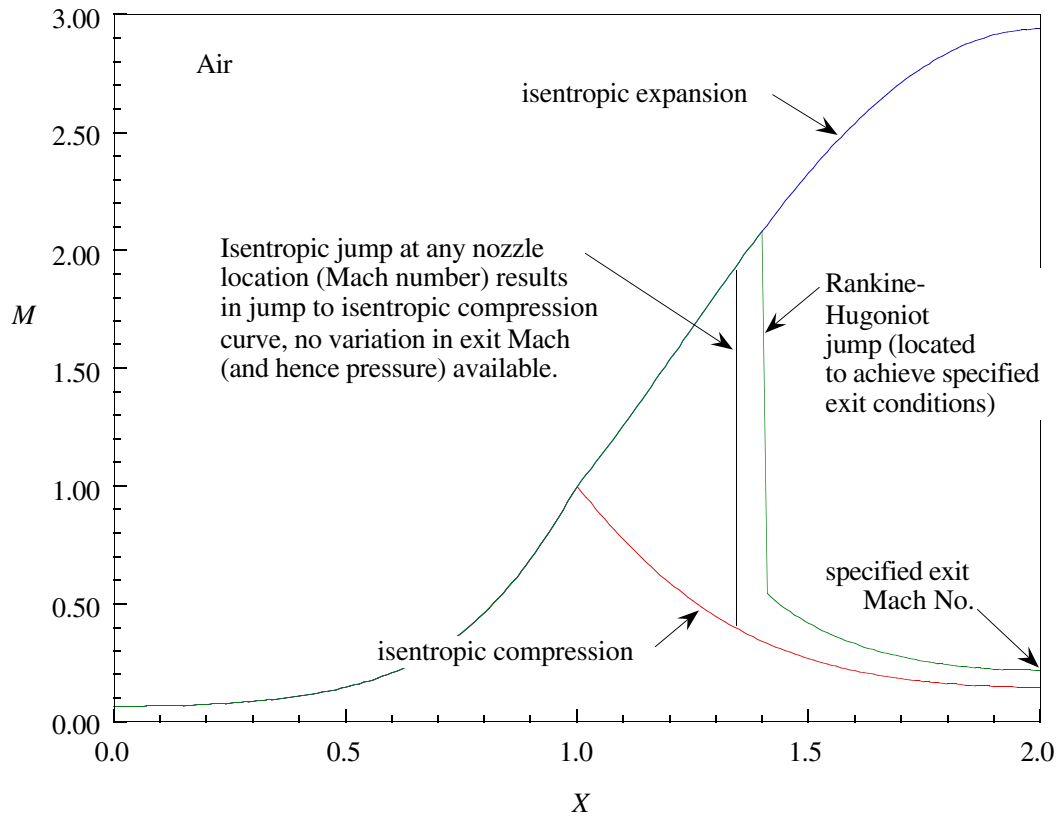


Fig. 4 Rankine-Hugoniot and isentropic shock jump implications for quasi-one dimensional nozzle flow.

Conclusion

The Laitone ratio reveals a fundamental difference between the potential and Euler flow models. Although the difference between the Rankine-Hugoniot and isentropic Mach jumps does not change drastically with Mach number, the Laitone pressure ratio does show a major change. The potential flow model results in the removal of a fundamental stability point in the flow.

The sudden breakdown in the potential flow approximation can be explained by two considerations. First, there is profound difference in the Laitone ratio, p_2/p_{01} , between Rankine-Hugoniot and isentropic jumps with increasing Mach number. Secondly, we have shown for the quasi-one dimensional nozzle flow model problem that the potential flow model cannot respond to the downstream pressure field through a change of shock position.

Finally, with increasing emphasis on multidisciplinary methods that frequently require thousands (or more) flow analyses, it appears worthwhile to reconsider the use of corrected potential flow models. Experience with Euler solutions has shown that the increase in model fidelity did not eliminate all the problems of obtaining robust and accurate numerical solutions.

Acknowledgements

The analysis presented here arose from work for the AC-SYNT Institute at Virginia Tech. We also acknowledge discussions with Mr. John Axelson and Dr. Bernard Grossman. Robert Narducci of Virginia Tech supplied the Euler solutions.

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