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Optimum Spanloads Incorporating Wing Structural Weight

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The classic minimum induced drag spanload is not necessarily the best choice for an aircraft. Here, a discrete vortex method which finds the minimum induced drag in the Trefftz plane has been used to calculate optimum spanloads for non-coplanar multi-surface configurations. The method includes constraints for lift coefficient, pitching moment coefficient and wing root bending moment. The wing root bending moment constraint has been introduced so that by holding wing geometry fixed, changes in wing weight can be related to variations in spanload distributions. Changes in wing induced drag and weight were converted to aircraft total gross weight and fuel weight benefits, so that the spanloads that give maximum take-off gross weight reduction can be found. Results show that a reduction in root bending moment from a lift distribution that gives minimum induced drag leads to more triangular spanloads, where the loads are shifted towards the root, reducing wing weight and increasing induced drag. A slight reduction in root bending moment is always beneficial, since the initial increase in induced drag is very small compared to the decrease in wing weight. Total weight benefits were studied for a B-777 type configuration, obtaining take-off gross weight improvements of about 1% for maximum range missions. When performing reduced-range missions, improvements can almost double. A long range, more aerodynamically driven aircraft like the B-777 will experience lower benefits as a result of increasing drag. Short to medium range aircraft will profit the most from more triangular lift distributions.

I. Introduction

The problem of finding the optimum lift distribution for a specific wing and aircraft configuration is difficult. Optimum spanloads that will give minimum induced drag for a given planform have been obtained since lifting line theory was developed by Prandtl. Planar wings were the main objective of this early work, although more advanced configurations (*i.e.* wing with winglets) were also treated. Recently, more advanced methods were developed that could deal with non-planar wings and multiple lifting surface configurations (*i.e.* a Vortex Lattice Method). These methods can find the lift distribution that gives minimum induced drag, generally performing the calculations in the Trefftz plane. However, the problem is more involved than this, since minimum drag will not be the only requirement for finding an optimum lift distribution. Indeed, the spanload also affects the wing structural weight. A general method for finding the optimum spanload should include both aerodynamics and structures together. It is then a multidisciplinary problem, and it is the coupling between disciplines that makes the problem more difficult.

A number of studies have been made treating both aerodynamics and structures in the problem. Prandtl[1] was the first one to note that the spanload for minimum induced drag was not the "optimum" spanload, and he calculated analytically the lift distribution giving minimum induced drag with a constraint in integrated bending moment[§] for planar wings. R.T. Jones[2] also performed analytical calculations for planar wings, using a root bending moment constraint. Later, Jones[3] studied minimum induced drag for wings with winglets using the same integrated bending

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[§] The classical structural models used as constraints for spanload calculations are a root bending moment constraint (the area under the load curve) and an integrated bending constraint (the area under the bending moment curve). Prandtl used this last constraint in his work.

moment constraint that Prandtl used for his analysis. Klein and Viswanathan,[4,5] combined both constraints of integrated and root bending moment and solved analytically for the optimum spanload for a given lift. More recently, numerical approaches have also been developed, such as those of Kroo[6], McGeer[7], and Craig and MacLean[8]. Kroo developed a computer program to optimize spanloads for arbitrary multiple lifting surface configurations. His program optimized induced drag for a given total lift, wing weight and trim. McGeer used an iterative scheme to find the optimum spanload for minimum drag with a fixed wing weight and parasite drag, including aeroelastic effects. Craig and McLean further introduced aeroelastic effects and fuselage interactions using the theory developed by Gray and Schenk[9], and studied wing weight and total drag (including profile drag) trade-offs. Kroo, and Craig and McLean employed more advanced structural models than a bending moment constraint. All the authors mentioned above let the span vary while maintaining a fixed wing weight, regardless of the structural model they used. Numerous other MDO methods have studied the combined aero-structures problem, although not addressing the spanload choice explicitly, *e. g*., Wakayama and Kroo.[10]

The past studies have established a few key results. For example, for a cantilever wing, keeping the wing weight fixed and increasing the span will produce a shift from the elliptical loading towards a more triangular one that, while maintaining the total lift constant, will reduce induced drag due to the increased span. However, a general conclusion about which spanloads are optimum has not yet been established. In particular, our interest is in the lift distribution for a fixed planform. This is the problem that would be incorporated explicitly in an MDO process for a complete aircraft.

The difficulty in finding optimum spanloads comes from the analysis carried out by the authors above. First of all, varying the span while keeping the wing weight fixed is not helpful when trying to compare lift distributions for a given planform, since comparisons will then be made for essentially different wings. A new approach is needed, in which the spanload is varied but the wing planform is not, so that the wing weight varies with the spanload. Secondly, the analysis must consider the entire configuration. The multidisciplinary nature of the problem should not be reduced to wing structures and wing drag, since the optimum spanload will bring the maximum benefit to the entire system, not just to the wing. When the lift distribution is varied, it is not only necessary to know how it will affect induced drag and wing weight, but how it will change the fuel and gross weights. Taking gross weight as the key measure of effectiveness (certainly better than either wing weight or induced drag) will help to find the optimum lift distribution that produces the maximum weight reductions.

In this paper the problem is treated through a perspective that will allow the designer to differentiate between spanloads that will be beneficial to the complete aircraft configuration and those that will not produce an improvement.

II. General Approach.

The lift distributions are obtained using a computer program (a general description of this program is given below) which optimizes the spanload of multiple lifting surface configurations with a constraint in wing root bending moment. This is the simplest structural constraint that can be imposed on the problem. It may appear that the approaches of Kroo[6] and Craig and McLean[8] are better, since they used more advanced structural models. This would be true if the wing weight were assumed to be a simple function of root bending moment. Instead, a general functional relationship is assumed of the form:

$$W_{wing} = f(root\_bending\_moment,...) \qquad (1)$$

The root bending moment is used only as a constraint for generating spanloads. It is not really the structural model used for the wing weight computation. Another important consideration comes from the fact that the interest is centered in changing wing weight while keeping the planform shape a constant. This is achieved through a special implementation of the root bending moment constraint in the aerodynamics code (see section III).

The structural model will be described in section IV. For now it is sufficient to say that given the load distribution (calculated using the wing root bending moment constraint) and the planform characteristics of the configuration, the model calculates the wing weight. Note that the aircraft gross weight is needed for this calculation.

Finally, once a new spanload is found, corresponding to a new induced drag coefficient and wing weight, it is necessary to study their effects on fuel and total gross weights. The Breguet range equation is used to find this effect. The description of how the Breguet equation is implemented in the program is given below.

If fuel weight and gross weight variations can be obtained for different lift distributions, the task of finding the optimum spanload will then be a matter of choosing which one is the best measure of effectiveness for a specific optimization problem. Choices include wing weight, fuel weight, gross weight, or a prescribed combination of each.

III. Description of the aerodynamics code.

The method used for calculating the lift distributions is a discrete vortex method with a Trefftz plane analysis. It was developed based on previous work by Blackwell[11], Lamar[12], Kuhlman[13] and Kroo.[6]. It determines the lift distribution corresponding to the minimum induced drag of the configuration. The implementation by Grasmeyer[14] in a FORTRAN code (*idrag* version 1.1) was used. The code also includes an optional trim constraint, in which the pitching moment coefficient can be fixed if several surfaces are analyzed. Given the geometry for a number of surfaces, the program finds the spanload that has the minimum induced drag for a specific value of lift coefficient and pitching moment coefficient using the method of Lagrange multipliers[14].

The code was modified to implement an extra constraint: a root bending moment constraint. Then, a new strategy was implemented in the code to obtain lift distributions with this constraint. First, the spanload for minimum induced drag is found without taking into consideration any bending moment constraints (*i.e.* for a planar wing, this will give an elliptic lift distribution). Once this spanload is obtained, the root bending moment it produces is calculated. This spanload and wing root bending moment will correspond to one unique value of wing weight. Next, the root bending moment is reduced by an arbitrary amount, say for example 10%, and a new optimum spanload is calculated with the same lift and the reduced wing root bending moment constraint. In that way, for a given planform, and knowing the relationship between weight and root bending moment[¶], the reduction in weight can be compared to the increase in induced drag.

[¶] The spanload generated with the root bending moment constraint is used to calculate wing bending material weight using a beam theory analysis.[15] Weight equations from FLOPS[16] are used for the other weight components. A detailed description of this weight calculation is given in section IV.

The bending moment constraint has been implemented in the *idrag* code using the method of Lagrange multipliers, and this constraint can also be turned on or off in combination with the trim constraint. Several configurations have been studied with this method. One of them is shown in **Figure 1**, a typical transport aircraft planform.

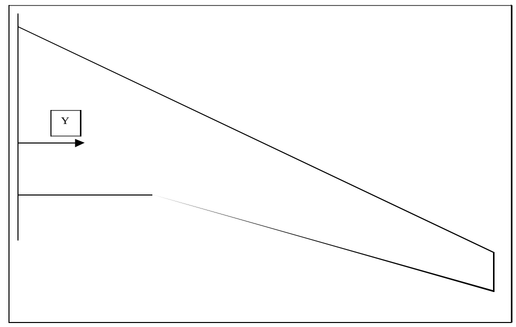

Figure 1. Typical transport aircraft planform.

Figure 2 shows two minimum induced drag spanloads for this planform at the cruise condition. One has no bending moment constraint, while the other is for an arbitrary wing root bending moment reduction (11%). Trimmed flight has been assumed, so that both bending moment and trim constraints are active.

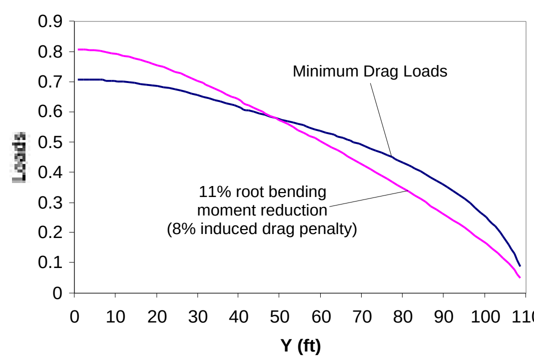

Figure 2. Span load distribution for minimum induced drag and with an arbitrary root bending moment reduction for a B-777 type configuration.

In **Figure 2** a reduction in wing root bending moment shifts the load curve towards a more triangularly loaded wing with the same lift. Since the load is shifted inwards, wing weight should be reduced with this new loading, and induced drag will necessarily be increased since the spanload deviates from the minimum induced drag distribution. In this case the drag increase from the minimum drag value is about 8%. One other consideration is that as the load curve becomes more triangular, the lift coefficient at the wing root also becomes larger for the same total wing lift coefficient. For a very high load near the wing root, stall at this location will necessarily occur at a lower wing lift coefficient value compared to that for a more elliptically loaded wing. Then, although it will be shown that shifting the lift distribution towards a more triangularly loaded wing will in most cases reduce the gross weight, there will be a limit on "how triangular" the spanload really can become based on stall considerations. In this study no limits based on stall are imposed on the optimization, and this will be valid as long as the spanload does not become "too triangular", with a high load value at the wing root. Recall that here the planform shape is constant. In a more general optimization problem the chord distribution would be optimized to account for stall behavior.[10]

IV. Structural Model

The structural model used for the key wing weight calculations was developed by Naghshineh-Pour[15] at Virginia Tech. Both cantilever and strut-braced wings can be handled with this model. The model was implemented in a computer code and has been validated as a realistic structural model for preliminary design. Taxi bump load analysis can be performed in the code, but is not used in this study. A maximum load factor condition will be the analysis condition used for wing weight calculations. The code requires the wing planform shape, together with wing thickness distributions and the spanload distribution for the maximum load factor as inputs. It gives the wing bending material weight as one output. The spanload for maximum load factor was obtained with the aerodynamic code. A value of 2.5 with a safety margin of 1.5 was the load factor used, since this is typical for transport aircraft. The required bending material weight along a variable box beam is calculated by integrating over the area under the bending moment curve. Engine inertia relief factors are also included.

The spanload corresponding to the maximum positive load is assumed to have the same shape as the load distribution for cruise. Thus a rigid wing is used in this paper and no aeroelastic effects are accounted for. In real life, the aeroelastic effects will cause a change in the load distribution from normal cruise conditions to the worst-case load conditions.

The bending material weight is then used to calculate the total wing weight together with equations taken from the general optimization code FLOPS[16] (Flight Optimization System), developed by NASA.

The wing weight calculations involve factors for wing sweep, wing area, flap area, aeroelastic tailoring factors and another factor accounting for the amount of composite materials used in the wing. The final weight equation used in the FLOPS subroutine is given here because it will become important when the Breguet range equation is introduced:

$$W_{wing} = \frac{TOGW \ \ w_1 + w_2 + w_3}{1 + w_1} \qquad (2)$$

In this equation: *GW* is the aircraft gross weight, w_1 is a factor that accounts for bending material weight, planform shape, ultimate load factor, engine inertia relief and aeroelastic and composite factors. It is the value supplied by the Naghshineh-Pour code. w_2 is an extra correction due to flap area, and w_3 further corrects wing weight for the amount of composites used.

Equation (2) was introduced here to show that the wing weight calculation requires the aircraft total gross weight. The purpose of this study is to change the lift distribution and observe how induced drag and wing weight change. This in turn will affect total fuel and gross weights. However, if the wing weight is dependent on aircraft gross weight, something must be done to close the loop. The use of the Breguet equation will not only serve as a method for calculating fuel and gross weights from a knowledge of induced drag and wing weight, but closes the loop connecting gross weight and wing weight.

V. Implementation of the Breguet equation.

The Breguet range equation is used as a means of relating wing weight and induced drag for a certain aircraft configuration to fuel and gross weights. Given the gross weight of a transport aircraft for a specified mission and the fuel weight and range for that same mission, the lift coefficient, drag coefficient and specific fuel consumption at the mission altitude and velocity can be calculated. The induced drag and lift distribution for minimum drag can also be obtained together with the corresponding wing weight. Once these calculations are performed, the following assumptions are made: the weight of the aircraft that does not include wing and fuel weight will remain constant, and the drag of the aircraft that does not include induced drag will also remain fixed. The validity of the results obtained in this study will depend on the validity of these assumptions. As for the first one, the structural weight of the whole aircraft will depend on wing weight and fuel weight, so that if these weights change, the structural weight should also change. However, for small variations in wing and fuel weight, that is, small changes in aircraft gross weight, it is not a bad assumption to presume that the rest of the structural weight (and of course, the payload weight) remains constant. In any event, if gross weight is reduced the structural weight should decrease. Hence, the gross weight reductions obtained here will be the minimum expected reductions. In the second assumption, it is presumed that changing the twist or camber distribution of the wing (recall that the planform remains the same) does not change profile or pressure drag. It seems clear that drag will change, but as long as the aircraft is still at the same speed, altitude, and lift coefficient conditions, the effects of the changing profile and pressure drag on the wing will be minor.

Once we assume that the only weight changes are wing and fuel weights, and that the only drag change is induced drag, the spanload can be modified to see the variation it produces on aircraft weights. The wing root bending moment is then decreased from the initial minimum drag configuration, producing a more triangularly loaded lift distribution with an increased value of induced drag. The new loads are then used in the structural model to calculate the wing weight. It was noted earlier that gross weight is needed to calculate wing weight from the FLOPS weight equation (2). Assuming that the mission range is constant (that is, the aircraft still has to meet the same mission requirements), all that is needed to calculate take-off weight from the Breguet range equation is the wing weight. Solving for the take-off weight from the Breguet equation gives:

$$TOW = (W_{wing} + W_{rest})\ e^{\frac{R\ sfc\ C_D}{V\ C_L}} \quad (3)$$

In this equation, W_{rest} is the weight that is assumed to remain constant, that is, the weight not covered by fuel or wing weight. For the maximum range configuration, where the aircraft has a full fuel load, it will be assumed that take-off weight equals aircraft gross weight. Combining equation (2) to calculate wing weight with equation (3) to calculate gross weight, it is clear that an iteration will yield simultaneously both the aircraft gross weight and wing weight. This is the approach we used. Now, a change in wing lift distribution towards a more triangular curve can be related to changes in wing weight, fuel weight and gross weight, so that an optimum lift distribution can be found.

When new gross weights are calculated corresponding to new lift distributions, the lift coefficient will change if the aircraft still flies at the same altitude, so that the loads used do not actually reflect in the calculated gross weights. Cruise altitude is then changed with gross weight variation to keep the cruise lift coefficient constant. In that way, the load distributions will correspond to weight variations. The actual altitude change needed to keep the lift coefficient constant is in fact small (less than 500 ft. for a Boeing B-777 class test case)

This is not the only approach that could have been used. Instead of varying gross weight and keeping mission range constant, the opposite could have been done. That is, the gross weight could be held constant, so that any wing weight reduction would be compensated by a fuel weight increase in the same amount, and the different ranges that different spanloads would produce could be compared. In the approach used here, fuel weight is calculated so that the required range is just met, and different gross weights are compared for the same mission. This approach corresponds to standard aircraft design practice.

VI. Results.

As an example of the method, a study has been performed for a Boeing B-777 type aircraft. The basic data used for the study is given in **Table 1**. Note that the configuration under study is a maximum take-off weight, maximum range, maximum fuel arrangement. Other missions for the same aircraft will generate different results. These other missions will be treated later. In our first example we concentrate on the maximum range configuration. As wing the root bending moment is decreased and new spanloads are calculated, the induced drag increases and the wing weight decreases. To compare induced drag coefficient increase to wing weight reduction as the root bending moment decreases, nondimensional parameters for both variables are defined as:

$$drag\_increase = 100 \times \frac{C_{Dind} - C_{Dind_0}}{C_{Dind_0}} \quad (4)$$

$$W_{wing}\_reduction = 100 \times \frac{W_{wing_0} - W_{wing}}{W_{wing_0}} \quad (5)$$

Here, the subscript “0” refers to the baseline configuration, “ind” refers to induced drag, and the subscript “wing” refers to the wing weight.

Table 1. B-777-type data used in the study.

WING GEOMETRY	
Location of Wing Chord Break Point	0.37
Wing Half Span (ft)	109.21
Inboard Wing Sweep (deg)	28.29
Outboard Wing Sweep (deg)	28.29
Wing Dihedral Angle (deg)	6
Wing Chord at Fuselage Center Line (ft)	52
Wing Chord at Break Point (ft)	25.83
Wing Chord at Wing Tip (ft)	7.35
Thickness to Chord at Fuselage Center Line (ft)	0.111
Thickness to Chord at Wing Break Point (ft)	0.1
Thickness to Chord at Wing Tip (ft)	0.08
HORIZONTAL TAIL GEOMETRY	
Horizontal Distance from wing to tail leading edges at center line	120
Vertical Distance from Wing to Tail Leading Edges at Center Line	12
Tail Half-Span (ft)	36.913
Tail Sweep Angle (deg)	37
Tail Dihedral Angle (deg)	0
Tail Root Chord (ft)	22.618
Tail Tip Chord (ft)	7.35
PERFORMANCE SPECIFICATIONS	
Maximum Gross-Weight (lbs)	588893
Fuel Weight (lbs)	215000
Maximum Range (nm)	7600 + 500 reserve
Cruise Mach Number	0.85
Cruise Altitude (ft)	40000
Static Specific Fuel Consumption (lb/hr/lb)	0.29
ENGINE SPECIFICATIONS	
Number of Engines on Wing	2
Engine Weight (lbs)	16278
Spanwise location of Engine on Wing	0.33
MISCELLANEOUS	
Ultimate Factor	2.5
Ratio of Wing Area Covered by Flaps	0.333
Location of Aircraft Center of Gravity (ft)	35
Fuselage Diameter (ft)	20.33

The results obtained for the B-777 class aircraft test case with the data given in **Table 1** are shown in **Figure 3**, in which induced drag increases and wing weight reductions are shown as a function of wing root bending moment reduction.

The induced drag curve shows that the drag increase from the minimum induced drag point (that is, zero root bending moment reduction) is parabolic, with zero slope at the starting point. This is expected, since it shows that the zero wing root bending moment reduction point is in fact the minimum for induced drag. The induced drag increase curve turns out to be exactly parabolic for every aircraft and planform configuration.

The wing weight reduction curve is much more interesting. Note the nearly linear behavior of this line. Even when a structural model was used that calculates wing bending material weight and takes into account such factors in the wing weight calculations as engine inertia relief or the use of composite materials, the variation of wing weight with wing root bending moment is almost linear.

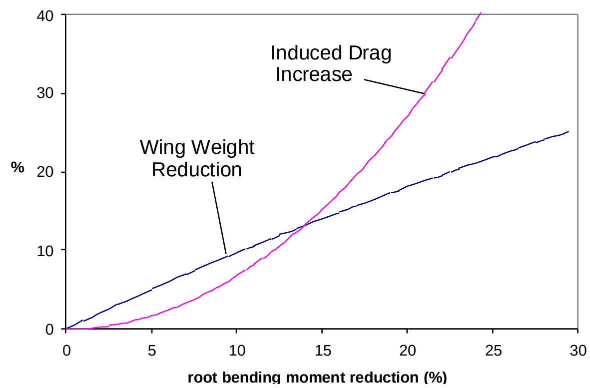

Figure 3. Wing weight reduction and induced drag increase versus root bending moment reduction. B-777 type aircraft, maximum range configuration.

We initially assumed that the wing root bending moment constraint was a somewhat crude approach. However, at least for this test case, it turns out to be a quite good one, and no further structural constraint may be needed.

It should be realized that the linearity of the wing weight reduction curve is in fact dependent on the test case, so that a different aircraft would yield a different curve. A wide variety of airplanes of different sizes, weights and mission ranges should be tested to prove that the linearity of the wing weight reduction versus root bending moment curve is in fact general, and that a straight line can be fit through this curve in all cases with a low loss in accuracy. If this is true, the entire spanload optimization only requires the study of two values for the wing weight and induced drag; one for minimum induced drag (zero root bending moment reduction) and another for an arbitrary root bending moment reduction. Curves would then be fit through these two points and the optimization process highly simplified, since it is no longer necessary to make calculations covering a wide range of root bending moment reductions.

Fuel weight variations versus root bending moment reductions for the same test case are shown in **Figure 4**. Fuel weight is actually decreased when the root bending moment reduction is low (from zero to six percent), even when the induced drag is increasing. This is due to the rapid decrease in wing weight compared to the increase in induced drag for these low values of root bending moment reduction, leading to a lighter aircraft with the same drag characteristics, so that less fuel is required to complete the mission. Nevertheless, when the root bending moment reduction becomes high, the fuel weight increases sharply, and this will in fact limit the value beyond which a more triangular spanload is beneficial.

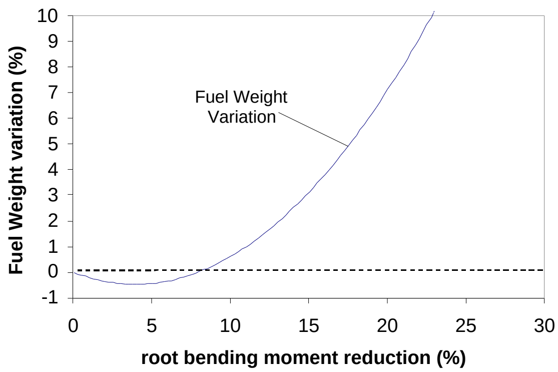

Figure 4. Fuel weight variation versus root bending moment reduction. B-777 type aircraft, maximum range configuration.

Figure 5 shows the variation of the sum of wing and fuel weights, also showing the variation of gross weight. These two curves are essentially the same plot, since the weight of the aircraft not covered by the wing and fuel weights was assumed to be constant. The minimum gross weight for the test case is found for a root bending moment reduction of about 10%. The spanload corresponding to this minimum gross weight is given in **Figure 6**, together with the lift distribution for minimum induced drag. Note that the maximum gross weight reduction obtained for this test case is close to 1%, or about 6,000 pounds. For a long range aircraft, like the B-777, any increase in drag results in a large increase in fuel weight, so that only a low wing root bending moment reduction would be beneficial. Thus, relatively small gross weight reductions are expected for this type of airplane. For low range transport aircraft, where structures become more important than aerodynamics, a larger root bending moment reduction will be optimum, and the total gross weight savings will be larger.

So far, the method presented gives the optimum spanload for a given aircraft. That is, the spanload that will produce a maximum reduction in gross weight. However, the test case studied was that of a maximum take-off weight configuration. Aircraft generally fly through shorter distances than their design maximum range, often in economic missions. It is then important to study how the spanloads generated by the root bending moment reduction will affect fuel weights and in turn take-off weights for these different missions. A modified FORTRAN code was then developed to carry out this study, the description of which is next.

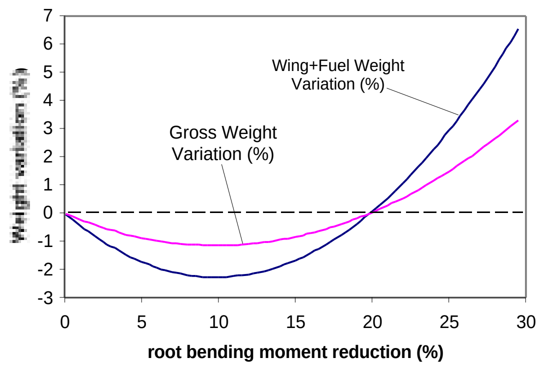

Figure 5. Wing plus fuel and gross weight variations versus root bending moment reductions. B-777 type aircraft, maximum range configuration.

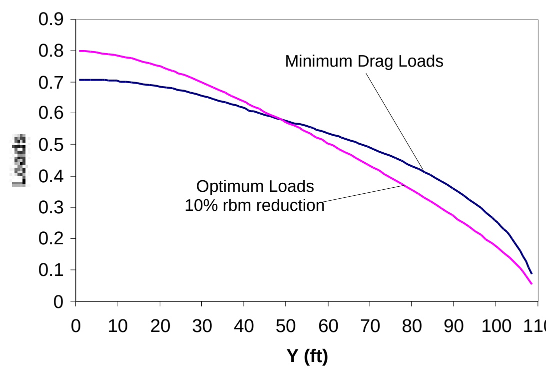

Figure 6. Span load distribution for minimum induced drag compared to optimum load distribution.

VII. Weight variations for different mission ranges.

The method developed to study weight variations for different mission ranges includes only a few modifications. Maximum take-off weight (equal to aircraft gross weight), maximum fuel weight and maximum range are still needed together with the new fuel weights for which weight variations will be studied. We assume that the initial take-off weight is reduced by the same amount from the maximum take-off weight than the fuel weight is reduced from the maximum fuel weight, that is, all the missions will have the same payload.

A new value of cruise lift coefficient for the new weights is calculated. It is then assumed that the drag coefficient not due to induced drag is still constant, with the same value it had for the maximum range configuration. Note that this assumption is really more restrictive than the one made before for the drag coefficient, since the cruise lift coefficient now changes, and consequently the angle of attack, which will change the drag coefficient. However, the mission is still performed at the same altitude and speed, and the lift coefficient varies little from the initial maximum weight configuration, so that drag coefficient will remain almost unchanged by the new conditions.

The range for the new fuel weight can be calculated using the Breguet equation, and different spanloads for different root bending moment reductions can be generated. For a given root bending moment reduction, the new induced drag for the new lift distribution is calculated as before, but the approach that was used to calculate wing weight can no longer be applied.

Using an iteration between the Breguet equation solved for the take-off weight (eqn (3)) and the wing weight equation from FLOPS (eqn (2)) will lead to inconsistent results. The take-off weight calculated in this iteration will serve as the gross weight used in the wing weight calculations, and for a reduced range mission take-off weight and gross weight will be different. Since for a specified wing root bending moment reduction the aircraft still has to meet the maximum range, the gross weights calculated for the maximum range configuration at the specific root bending moment reduction are used into the wing weight equation so that the iteration is removed and the calculated wing weights for different mission ranges are the same. The actual take-off weight is then calculated once the wing weight is known using equation (3), and fuel weights are found in the same way they were before.

One final consideration must be pointed out: take-off weight variations and fuel weight variations for different mission ranges will be non-dimensionalized by maximum weights for consistency. That is, the weight difference will always be divided by the fuel or take-off weight corresponding to the maximum range configuration. Otherwise, comparisons would be made for variations that are non-dimensionalized by different weights. Results for the B-777-class aircraft study case are given next.

VIII. Results for different mission ranges.

In this case the spanload is optimized for a reduced range, while the aircraft still meets the full, long range mission requirement as a constraint. In minimizing the take-off weight at a shorter range, a penalty will be incurred for the full range mission compared to the basic aero-alone optimum weight. The relative magnitude of the benefit for shorter range flight compared to the penalty at the full mission range is found as part of the results, and can be used in determining the best design target choice.

The study is performed reducing fuel weights so that ranges vary from 4000 to 8000 nautical miles, which are typical mission ranges for the B-777. Wing weights for different root bending moment reductions will have the same value as those obtained for the maximum range configuration, so that its variation will not be shown here.

Figure 7 shows the fuel weight variation as a function of wing root bending moment reduction for different mission ranges. All the curves appear close to each other when the root bending moment reduction is low, but for high values of root bending moment reduction (more triangular spanloads), the fuel weight needed to complete the mission range increases much more sharply for high mission ranges, and this is reflected in aircraft take-off weight.

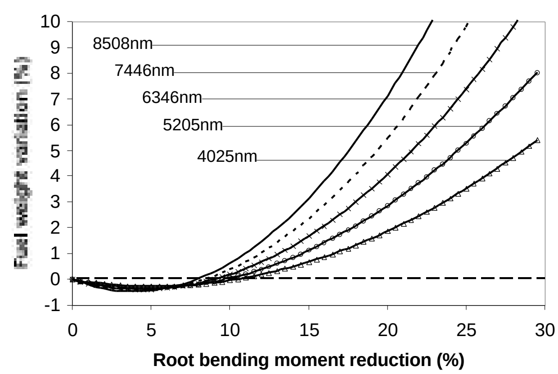

Figure 7. Fuel weight variation with root bending moment reduction for different mission ranges. B-777 type aircraft.

Figure 8 represents the variation of the sum of wing and fuel weights as a function of root bending moment reduction. **Figure 9** depicts take-off weight variation. Again figures 8 and 9 are essentially the same graph, since wing and fuel weights are the only weights that change in the model used. The general result is that a larger take-off weight reduction can be achieved for lower ranges, also corresponding to a higher root bending moment reduction and a more triangular lift distribution. This means that the take-off weight savings due to reduced wing weights become more important than the fuel weight increments due to increased drag for lower mission ranges, as expected.

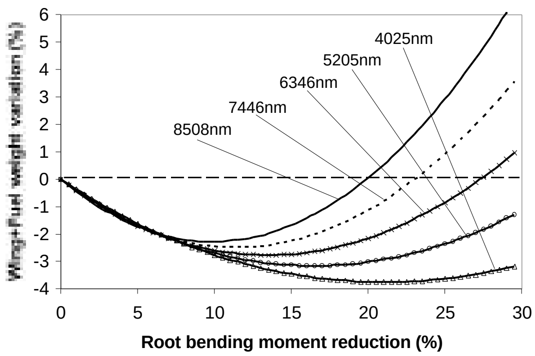

Figure 8. Wing plus fuel weight variation with root bending moment reduction for different mission ranges. B-777 type aircraft.

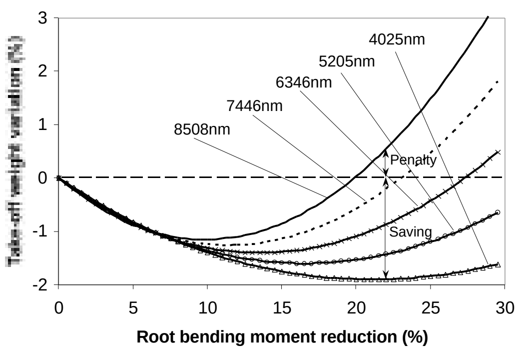

Figure 9. Gross weight variation with root bending moment reduction for different mission ranges. Boeing B-777 type aircraft.

It should also be realized that the optimum load distributions for short range configurations can result in weight penalties for the maximum range mission. Figure 9 shows the maximum take-off weight reduction for the 4025 nm. mission range case, corresponding to a 22% root bending moment reduction. Weight savings when performing this reduced mission range are around 2%. However, for the same spanload, if the maximum mission range is performed, a take-off weight penalty of about 0.5% will be experienced. It is then important to choose an optimum spanload that represents a compromise between weight savings for the different mission ranges that the aircraft will perform.

IX. Conclusions.

A method for calculating lift distributions for minimum induced drag with a wing root bending moment constraint, and that relates the associated spanloads to changes in wing weight, fuel weight and gross weight for transport aircraft configurations has been developed. The method can help determine which spanloads provide the maximum benefit to a specific aircraft design, so that an optimum lift distribution can be found.

The key insight is that the wing weight decreases nearly linearly with reduced wing root bending moment, while the additional induced drag arising from forcing the wing root bending moment to be less than its minimum drag value results in a parabolic increase in induced drag. Therefore, the system minimum will always occur for a spanload with a lower wing root bending moment than the minimum based on aerodynamics alone.

Even for the same airplane fuel weight, variations due to different lift distributions change from mission to mission, depending on the range, so that a wing spanload curve that will produce large benefits when performing a low range mission can result in a penalty when performing its maximum range mission. It is then necessary to study aircraft configurations through the whole range of missions they cover to find an optimum spanload that will represent a compromise for the different missions.

A B-777 class aircraft has been studied with this method. Results show that a reduction of 1% in maximum take-off gross weight can be obtained. When performing reduced range missions, an almost 2% savings in take-off weight savings is achieved.

The method described here is applicable to aircraft configurations with cantilever wings. It needs to be extended to treat the case of strut-braced wing concepts which have recently been shown to be very promising[17, 18].

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