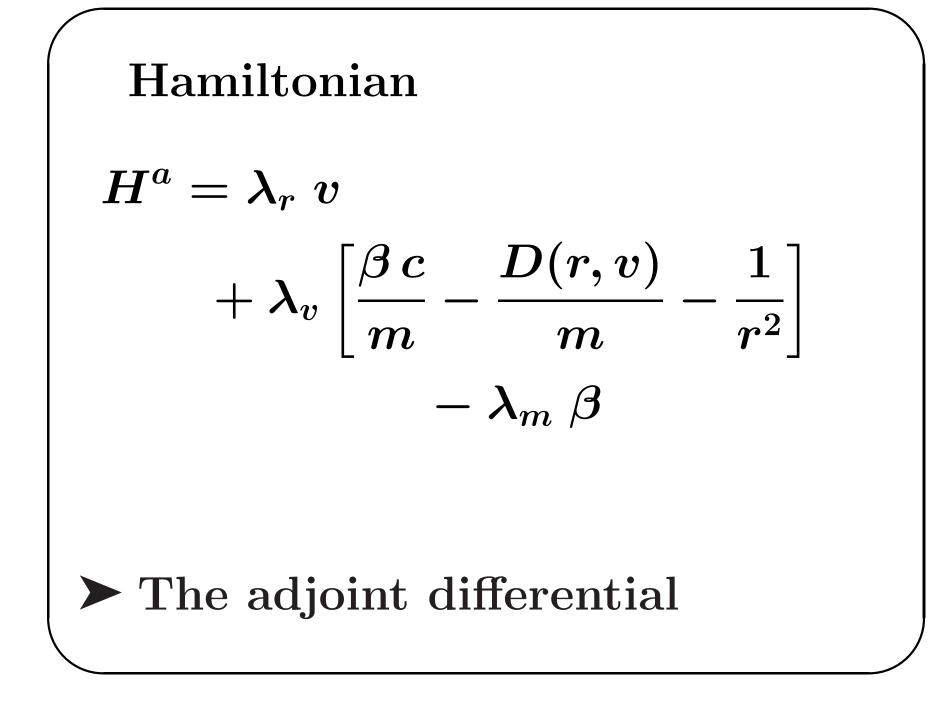
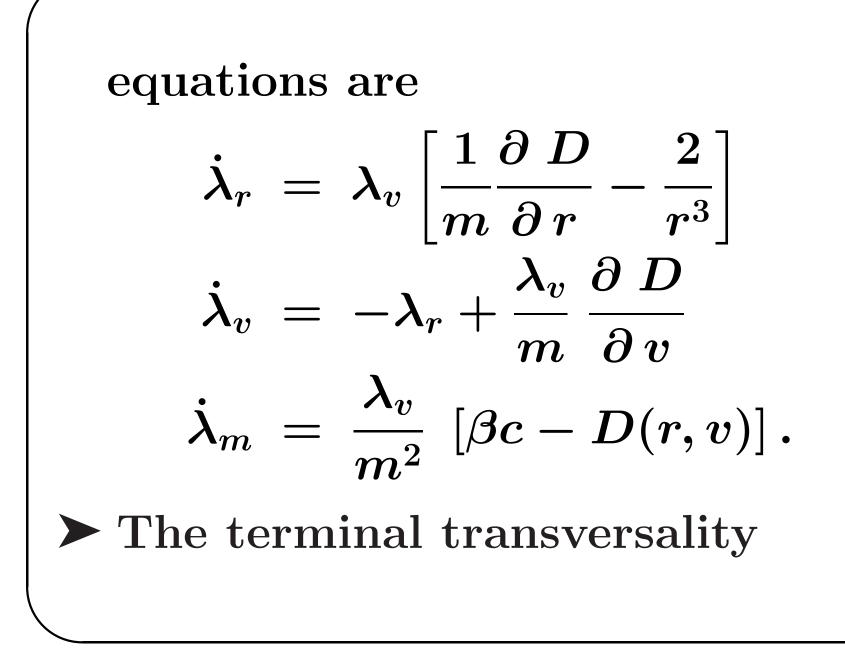


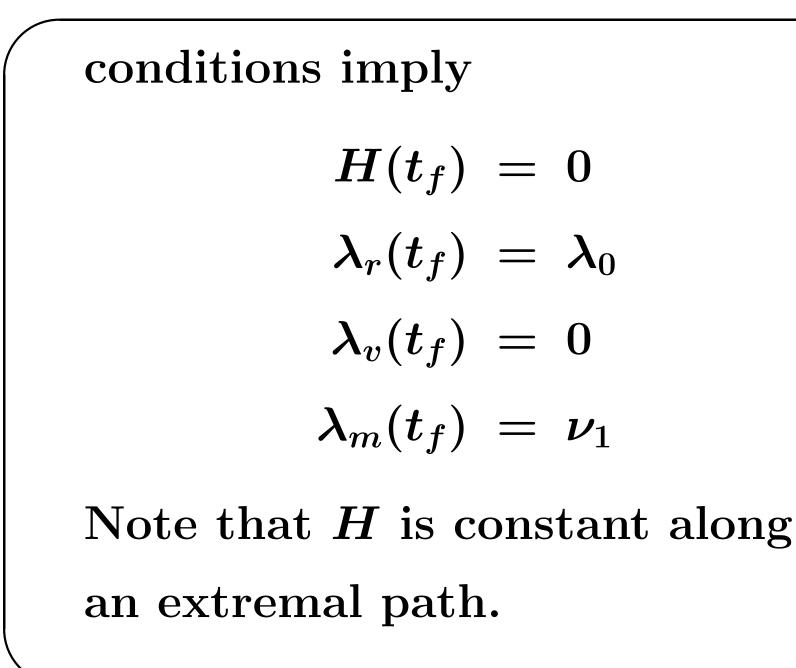
with $\tilde{\beta} \in [0, \tilde{\beta}_{\max}]$, where, for example, $ilde{r}=r/R_e$ ► In the following we drop the ~ and note that all quantities have been non-dimensionalized.

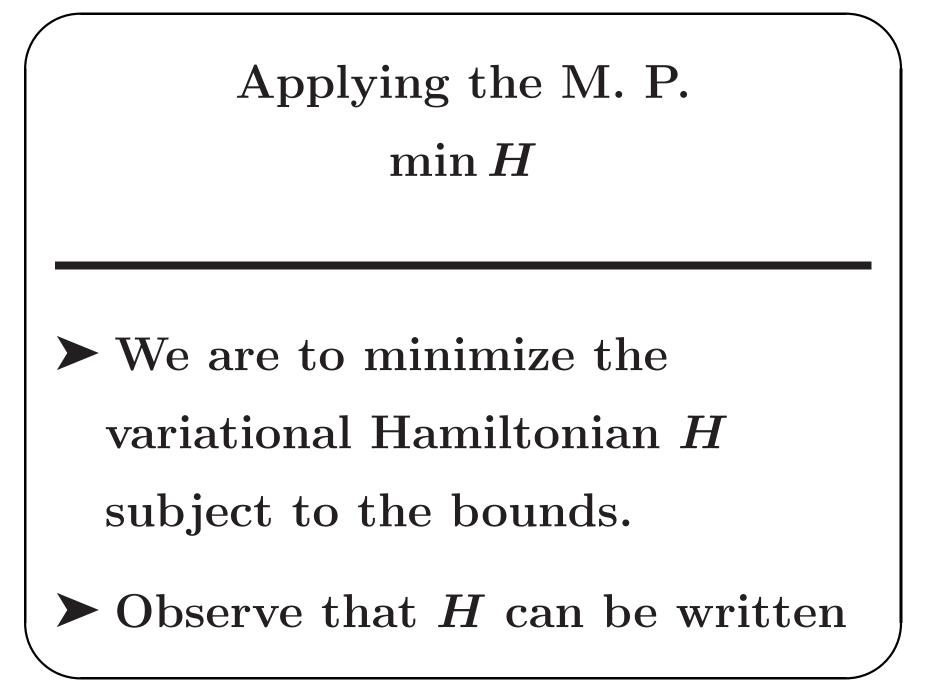
Applying the M.P.

► We form the variational

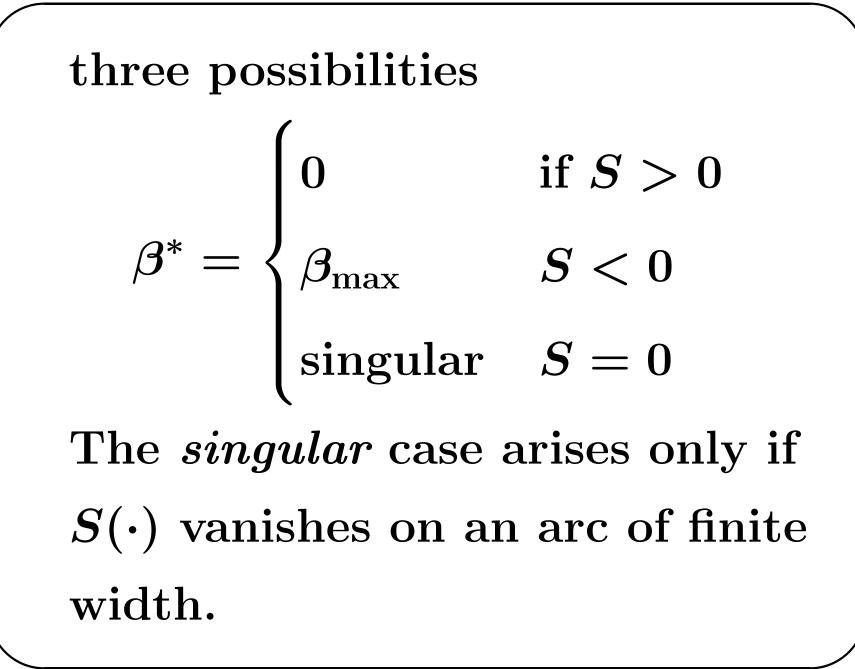




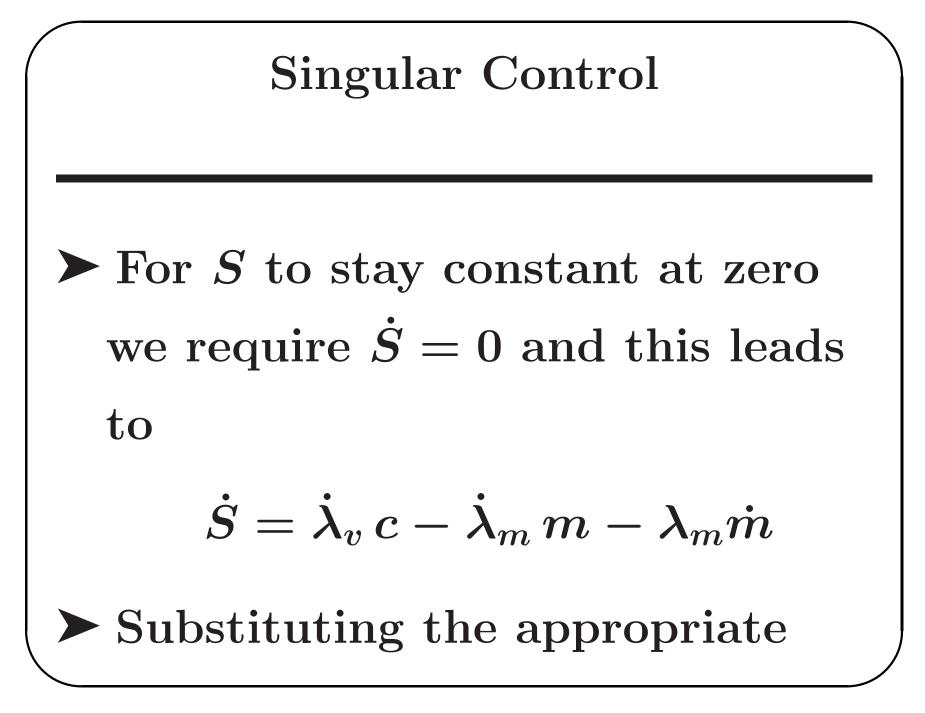




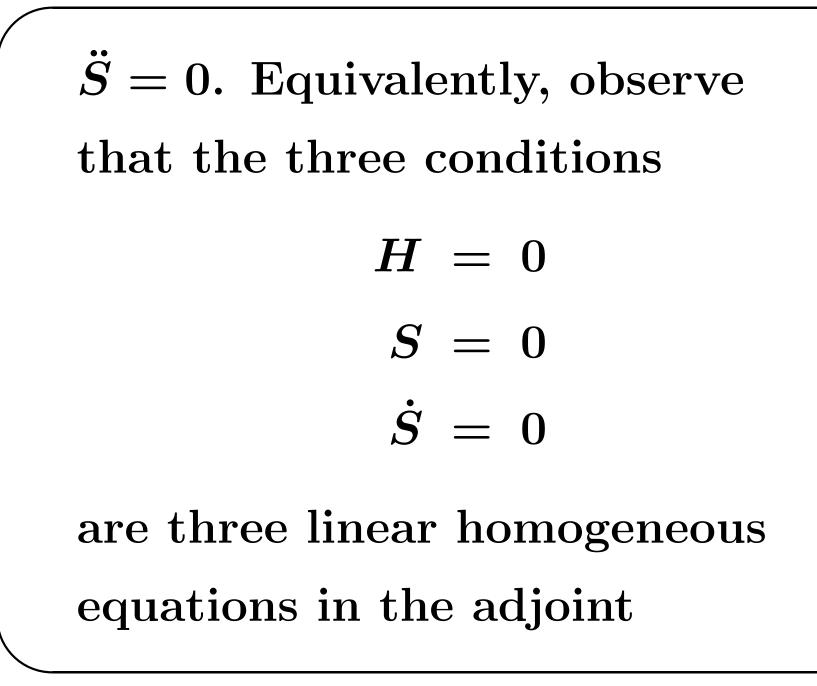
as $H = \left|rac{\lambda_v\,c}{m} - \lambda_m
ight|eta$ + terms independent of β Use the symbol S for the terms in square brackets For the mass flow-rate β we find



Since *m* is positive we can multiply *S* by *m* without changing the conclusions. Hence we re-define $S = [\lambda_v c - \lambda_m m]$



state/adjoint differential equations this simplifies to $\dot{S} = rac{\lambda_v}{m} \left| D + c rac{\partial \ D}{\partial \ v}
ight| - \lambda_r c$ Note that the β terms have cancelled out. ► We could take a second time-derivative and insist that



variables $\lambda_r, \lambda_v, \lambda_m$. Since the adjoints can not all vanish simultaneously, this implies that the determinant must be zero. That is, $v\left[D+crac{\partial \ D}{\partial \ v}
ight]-c\left[D+rac{m}{r^2}
ight]=0$

► Since this involves only state variables it's somewhat simpler than \dot{S} . Setting the time-derivative of this expression to zero will lead to an expression for the *singular* control β . It is still somewhat messy.

► In general, the control will appear first in an even time-derivative of the switching function. If the control appears first in the 2q-th time-derivative of S, we say the singular arc is of order q. The Goddard problem has a first order singular arc.